PROMETHEE & GAIA methods

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Short Bio

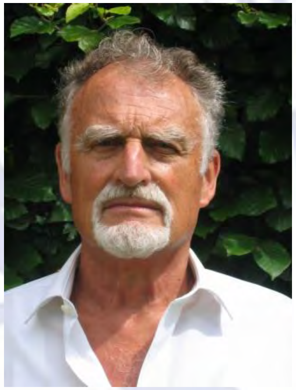
- Degree in Mathematics (ULB, 1998), consultant in Risk Management (1998→2000), DEA in Applied Sciences (ULB, 2001), PhD in Applied Sciences (ULB, 2005);
- Associate Professor at the Polytechnic School since 2007;
- Head of the SMG unit (2007);
- Research interets:
  - Foundations, properties and applications of the PROMETHEE & GAIA methods
  - Multi-objective optimization (exact methods and heuristics)
  - Trans-disciplinary aspects of MCDA (MCDA & GIS, & Clustering, & Evidence theory, & DEA)
- Currently supervising 3 PhD students;
- Co-founder of the D-SIGHT spin-off: www.d-sight.com

Outline

- Introduction
- A pedagogical example
- PROMETHEE I & II rankings
- GAIA
- Software demonstration: D-SIGHT
- A few words about rank reversal
- Preference elicitation
- Conclusion

Historical background

Prof. Jean-Pierre Brans
(VUB, Solvay School)



Prof. Philippe Vincke
(ULB, Engineering Faculty)

Prof. Bertrand Mareschal
(ULB, Solvay Brussels School of Economics and Management)

Applications

Behzadian, M.; Kazemzadeh, R.B.; Albadvi, A.; Aghdasi, M. (2010) « *PROMETHEE: A comprehensive literature review on methodologies and applications* », *EJOR, Vol.200(1),198-215*

- > 200 papers published in > 100 journals
- Topics: *Environmental management, hydrology and water management, finance, chemistry, logistics and transportation, energy management, health care, manufactoring and assembly, sports,…*

Let us agree on a few points

- Multicriteria decision problems are ill-defined (no optimal solutions);
- Decision aid versus decision making;
- « *The purpose of models is not to fit the data but to sharpen the questions* », Samuel Karlin;
- Parameters value:
  - Interpretation;
  - robustness

Let us start with a educational example !

An educational example

- A plant location problem
  - 6 possible locations
  - 6 criteria

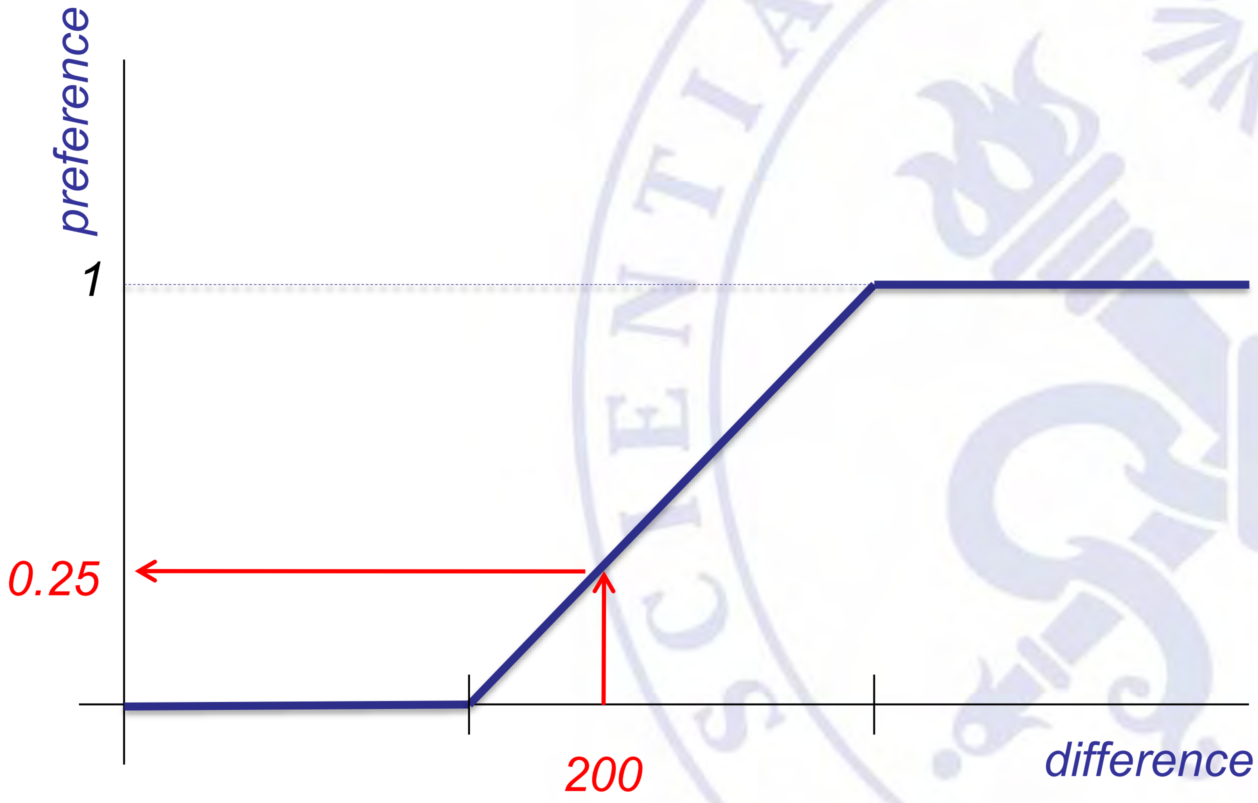
	Engineers	Power	Cost	Maintenance	Village	Security
Italy	75	90	600	5,4	**8**	5
Belgium	65	**58**	**200**	**9,7**	**1**	**1**
Germany	83	60	400	7,2	4	7
Sweden	**40**	80	**1.000**	7,5	7	**10**
Austria	52	72	600	**2**	3	8
France	**94**	**96**	700	3,6	5	6

Main principle: pair-wise comparisons

	Engineers	Power	Cost	Maintenance	Village	Security
Italy	75	90	600	5,4	**8**	5
Belgium	65	**58**	**200**	**9,7**	**1**	**1**
Germany	83	60	**400**	7,2	4	7
Sweden	**40**	80	**1.000**	7,5	7	**10**
Austria	52	72	**600**	**2**	3	8
France	**94**	**96**	700	3,6	5	6

- Concerning the cost, Germany is better than Austria !
- How can we quantify this advantage ? 200 ?
- What does it mean ?

Unicriterion preference function



Step 1: compute unicriterion preference degree for every pair of alternatives

	Engineers	Power	Cost	Maintenance	Village	Security
			0.25			
			-200			
Germany	83	60	400	7,2	4	7
Austria	52	72	600	2	3	8
	-31	12		-5.2	-1	1
	1	0.75		1	0.3	0.63

Step 2: compute global preference degree for every pair of alternatives

			0.25			
Germany	83	60	400	7,2	4	7
	Engineers	Power	Cost	Maintenance	Village	Security
? **Weights**	**0.2**	**0.2**	**0.2**	**0.1**	**0.15**	**0.15**
Austria	52	72	600	**2**	3	8
	0.5	**0.75**		**1**	**0.3**	**0.63**

*Π(Austria, Germany)=0.5*0.2+ 0.75* 0.2 +1*0.1+0.3*0.15+0.63*0.15*
= 0.489

Π(Germany,Austria)= ***0.25*0.4=0.05***

Preference matrix

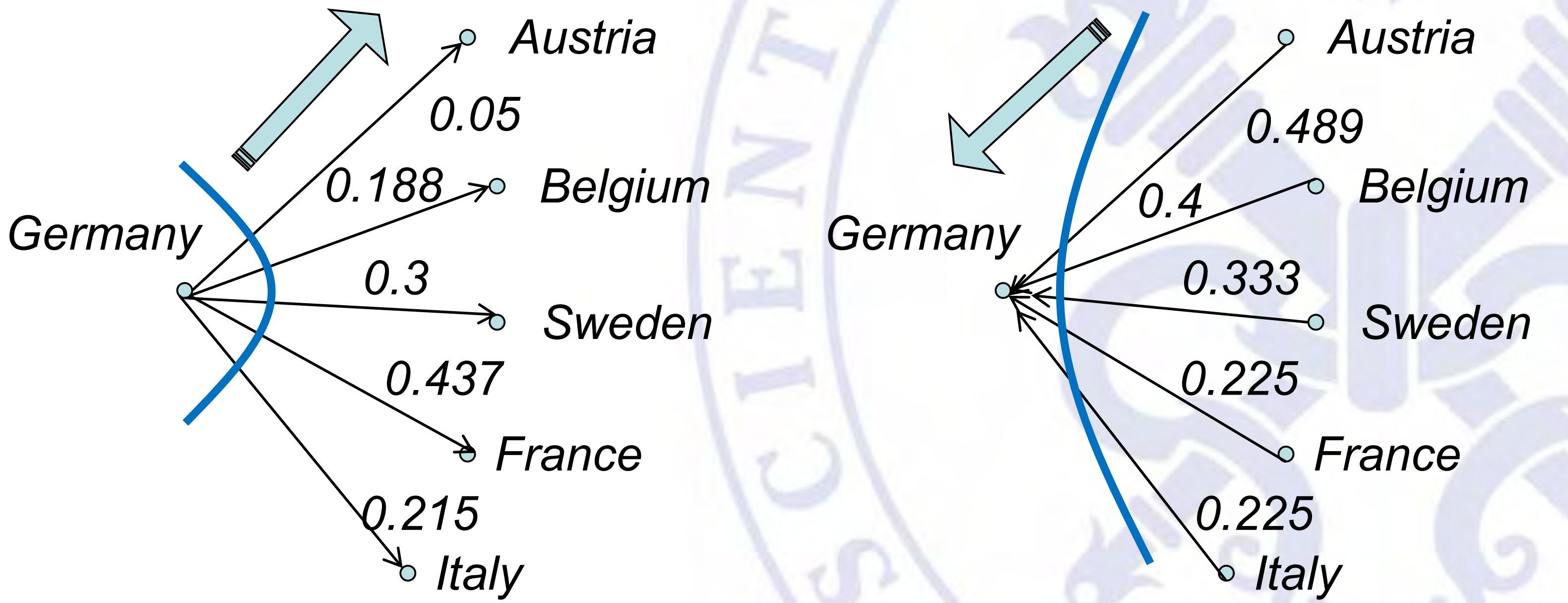
Preference Degrees

Preference Degrees

	Italy	Belgium	Germany	Sweden	Austria	France
Italy	0,000	0,280	0,225	0,242	0,090	0,217
Belgium	0,267	0,000	0,400	0,300	0,267	0,500
Germany	0,215	0,188	0,000	0,300	0,050	0,437
Sweden	0,429	0,545	0,333	0,000	0,255	0,255
Austria	0,458	0,545	0,489	0,342	0,000	0,457
France	0,259	0,379	0,225	0,388	0,120	0,000

- How can we exploit this matrix ?
- … in order to obtain a ranking (complete or partial) ?

Step 3: compute positive, negative and net flow scores



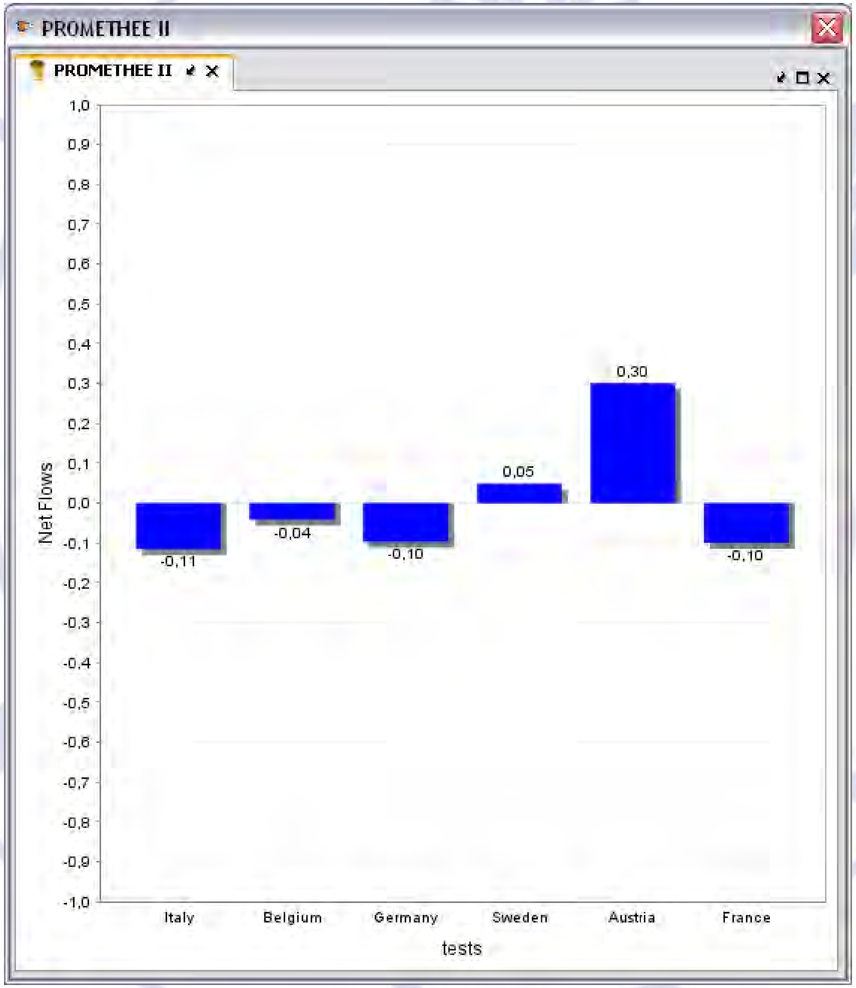
$\Phi^+(Germany) = 0.238$ $\Phi^-(Germany) = 0.334$

$\Phi(Germany) = \Phi^+(Germany) - \Phi^-(Germany) = -0.1$

PROMETHEE II

Flows

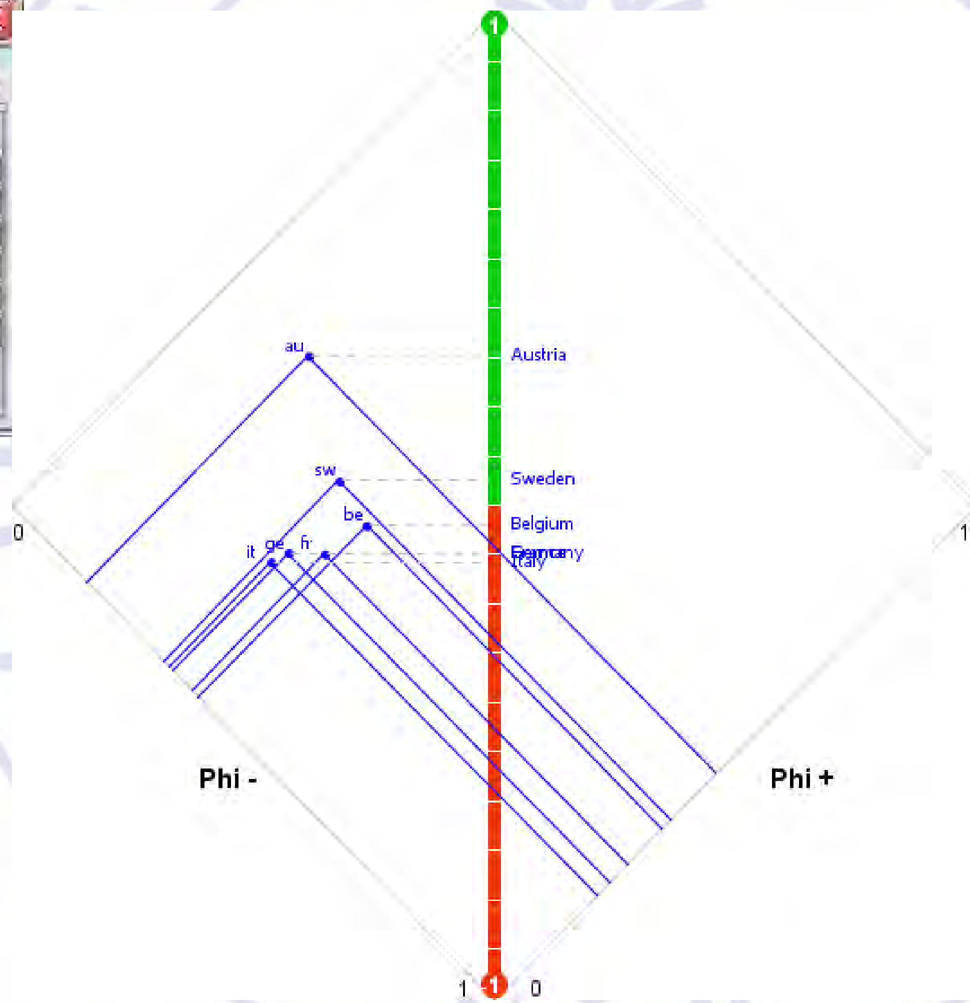
Alternative	Rank	Net Flow	Positive Flow	Negative Flow
Austria	1	0,302	0,458	0,156
Sweden	2	0,049	0,363	0,314
Belgium	3	-0,041	0,347	0,387
Germany	4	-0,096	0,238	0,334
France	5	-0,099	0,274	0,373
Italy	6	-0,115	0,211	0,326



PROMETHEE I

Flows

Alternative	Rank	Net Flow	Positive Flow	Negative Flow
Austria	1	0,302	0,458	0,156
Sweden	2	0,049	0,363	0,314
Belgium	3	-0,041	0,347	0,387
Germany	4	-0,096	0,238	0,334
France	5	-0,099	0,274	0,373
Italy	6	-0,115	0,211	0,326



Formalization

PROMETHEE

Preference Ranking Organisation METHod for Enrichment Evaluations

Formalization

- A finite set of alternatives:

$$A=\{a_1,a_2,..,a_n\}$$

- A set of criteria:

$$F=\{f_1,f_2,..,f_q\}$$

- W.l.g. these criteria have to be maximized

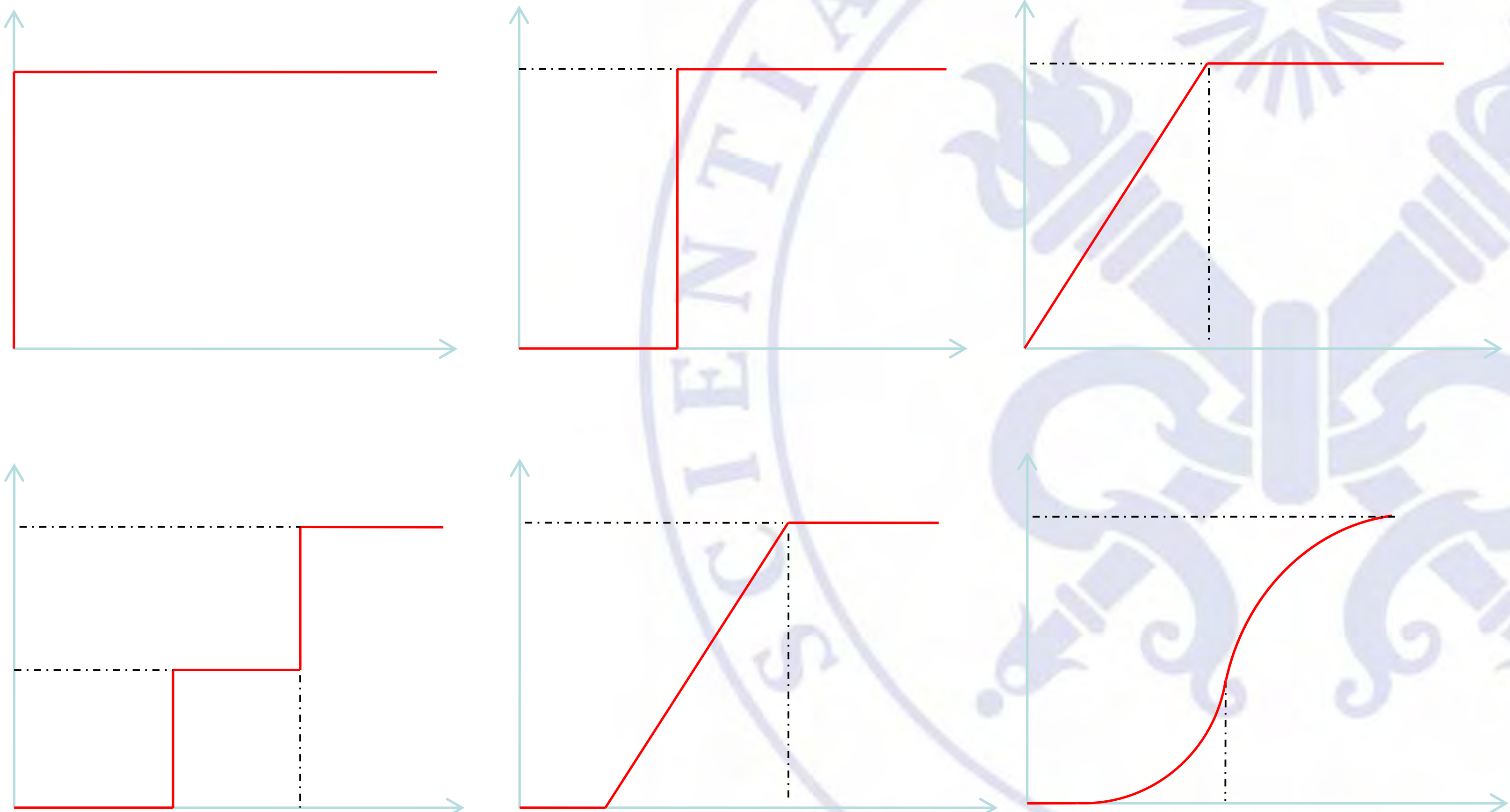
Step 1: uni-criterion preferences

$$\forall a_i, a_j \in A : d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)$$

$$\pi_k(a_i, a_j) = P_k[d_k(a_i, a_j)]$$

This operation has to be meaningful:
<u>INTERVAL SCALE</u>

Preference functions



Step 2: Compute preference matrix

$$\forall a_i, a_j \in A : \pi(a_i, a_j) = \sum_{k=1}^{q} w_k \pi_k(a_i, a_j)$$

As a consequence:

$$\pi(a_i, a_i) = 0$$

$$\pi(a_i, a_j) \geq 0$$

$$\pi(a_i, a_j) + \pi(a_j, a_i) \leq 1$$

Step 3: compute flow scores

$$\phi^{+}(a_i) = \frac{1}{n-1} \sum_{b \in A} \pi(a_i, b)$$

$$\phi^{-}(a_i) = \frac{1}{n-1} \sum_{b \in A} \pi(b, a_i)$$

$$\phi(a_i) = \phi^{+}(a_i) - \phi^{-}(a_i)$$

Maximum number of parameters: $\boxed{3.q-1}$

PROMETHEE II

Complete ranking based on the net flow score.

$$a_i P a_j \Leftrightarrow \Phi(a_i) > \Phi(a_j)$$

$$a_i I a_j \Leftrightarrow \Phi(a_i) = \Phi(a_j)$$

PROMETHEE I

Partial ranking based on both the positive and negative flow scores.

$$a_i P a_j \Leftrightarrow [\phi^+(a_i) > \phi^+(a_j)] \wedge [\phi^-(a_i) \leq \phi^-(a_j)]$$

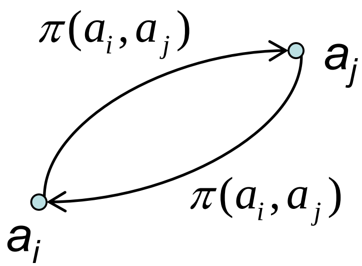
$$a_i P a_j \Leftrightarrow [\phi^+(a_i) \geq \phi^+(a_j)] \wedge [\phi^-(a_i) < \phi^-(a_j)]$$

$$a_i I a_j \Leftrightarrow [\phi^+(a_i) = \phi^+(a_j)] \wedge [\phi^-(a_i) = \phi^-(a_j)]$$

$$a_i J a_j, otherwise$$

The net flow score: a recipe ?

- From <u>local</u> to <u>global</u> information !



$S_i = S(a_i), S(a_j) = S_j$

ill-defined problem

- One could expect that:

$$\pi_{ij} - \pi_{ji} \approx S_i - S_j$$

Property

Intuition: “The PROMETHEE multicriteria net flow $\phi(a_i)$ is the centred score s_i (i=1,…,n) that minimizes the sum of the squared deviations from the pair-wise comparisons of the actions”

$$Q = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left(s_i - s_j\right) - \left(\pi_{ij} - \pi_{ji}\right)\right]^2$$

Proof (1):

$$L\left(s_1,\ldots,s_n,\lambda\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left(s_i-s_j\right)-\left(\pi_{ij}-\pi_{ji}\right)\right]^2-\lambda\sum_{i=1}^{n}s_i$$

$$\frac{\partial L\left(s_1,\ldots,s_n,\lambda\right)}{\partial s_i}=0$$

$$\frac{\partial L\left(s_1,\ldots,s_n,\lambda\right)}{\partial \lambda}=0$$

Proof (2):

$$L(s_1,\ldots,s_n,\lambda)=\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left(s_i-s_j\right)-\left(\pi_{ij}-\pi_{ji}\right)\right]^2-\lambda\sum_{i=1}^{n}s_i$$

$$\frac{\partial L}{\partial s_i}=4.\sum_{j=1,\,j\neq i}^{n}[(s_i-s_j)-(\pi_{ij}-\pi_{ji})]-\lambda$$

$$=4.[(n-1).s_i-\sum_{j=1,\,j\neq i}^{n}s_j-\sum_{j=1,\,j\neq i}^{n}(\pi_{ij}-\pi_{ji})]-\lambda$$

$$=4[n.s_i-\sum_{j=1,\,j\neq i}^{n}(\pi_{ij}-\pi_{ji})]-\lambda$$

$$\boxed{s_i=\frac{1}{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\pi_{ij}-\pi_{ji})=\frac{n-1}{n}\phi(a_i)}$$

Preferential independance (1)

J $\subset$ *F is preferentially independent within G if* $\forall$ *a, b, c, d* $\in$ *A such that*

$$\boxed{\begin{aligned} f_j(a) &= f_j(b), \forall j \in J \\ f_j(c) &= f_j(d), \forall j \in J \\ f_j(a) &= f_j(c), \forall j \in \bar{J} \\ f_j(b) &= f_j(d), \forall j \in \bar{J} \end{aligned}}$$

We have $\quad a\,P\,b \;\Leftrightarrow\; c\,P\,d$

Preferential independance (2)

	Dish	Sauce
a	French fries	Bolognese
b	Spaghetti	Bolognese
c	French fries	Mayonnaise
d	Spaghetti	Mayonnaise

Prerential independance (3)

$$f_j(a) = f_j(b), \forall j \in J$$
$$f_j(c) = f_j(d), \forall j \in J$$
$$f_j(a) = f_j(c), \forall j \in \bar{J}$$
$$f_j(b) = f_j(d), \forall j \in \bar{J}$$

$$aPb \Leftrightarrow \phi(a) > \phi(b)$$

$$\phi(a) = \sum_{j=1}^{q} w_j.\phi_j(a) = \sum_{j \in J} w_j.\phi_j(a) + \sum_{j \in \bar{J}} w_j.\phi_j(a)$$
$$= \boxed{\sum_{j \in J} w_j.\phi_j(b)} + \sum_{j \in \bar{J}} w_j.\phi_j(a) > \boxed{\sum_{j \in J} w_j.\phi_j(b)} + \sum_{j \in \bar{J}} w_j.\phi_j(b) = \phi(b)$$

$$\sum_{j \in \bar{J}} w_j.\phi_j(a) > \sum_{j \in \bar{J}} w_j.\phi_j(b)$$

Prerential independance (5)

$$f_j(a) = f_j(b), \forall j \in J$$
$$f_j(c) = f_j(d), \forall j \in J$$
$$f_j(a) = f_j(c), \forall j \in \bar{J}$$
$$f_j(b) = f_j(d), \forall j \in \bar{J}$$

$$\sum_{j \in \bar{J}} w_j . \phi_j(a) > \sum_{j \in \bar{J}} w_j . \phi_j(b)$$

$$\phi(c) = \sum_{j=1}^{q} w_j . \phi_j(c) = \sum_{j \in J} w_j . \phi_j(c) + \sum_{j \in \bar{J}} w_j . \phi_j(c)$$
$$= \sum_{j \in J} w_j . \phi_j(d) + \sum_{j \in \bar{J}} w_j . \phi_j(a) > \sum_{j \in J} w_j . \phi_j(d) + \sum_{j \in \bar{J}} w_j . \phi_j(b)$$
$$= \sum_{j \in J} w_j . \phi_j(d) + \sum_{j \in \bar{J}} w_j . \phi_j(d) = \phi(d)$$

GAIA

Geometrical Analysis for Interactive Assistance

GAIA (1)

- We have:

$$\Phi(a_i) = \frac{1}{n-1}\sum_{b\in A}\sum_{k=1}^{q} w_k.\pi_k(a_i,b) - \frac{1}{n-1}\sum_{b\in A}\sum_{k=1}^{q} w_k.\pi_k(b,a_i)$$

$$= \sum_{k=1}^{q} w_k.\frac{1}{n-1}\sum_{b\in A}\pi_k(a_i,b)-\pi_k(b,a_i) = \sum_{k=1}^{q} w_k.\phi_k(a_i)$$

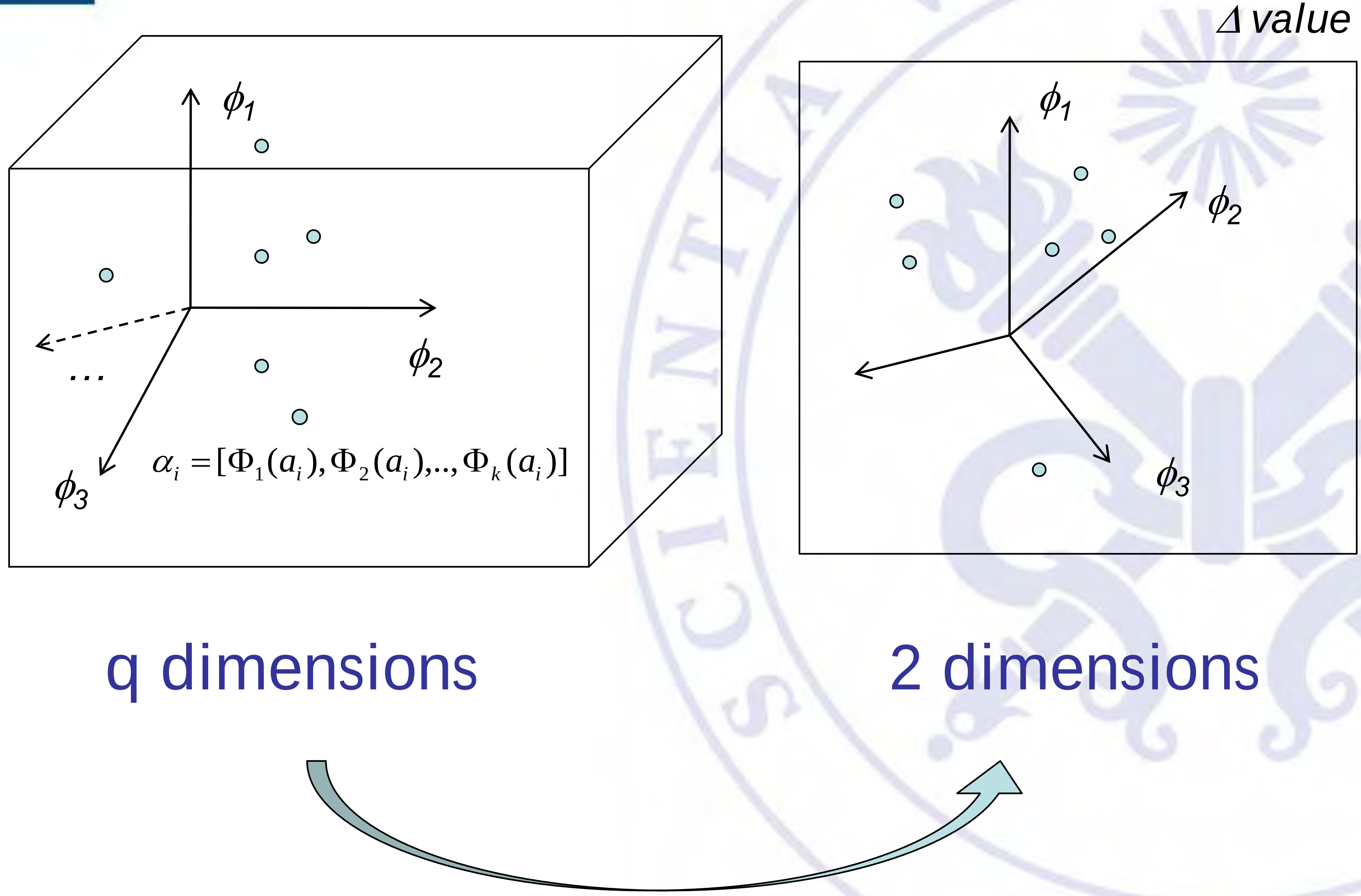
- Where

$$\Phi_k(a_i) = \sum_{b\in A}\pi_k(a_i,b)-\pi_k(b,a_i)$$

- In other words, every alternative can be represented by a vector:

$$\alpha_i = [\Phi_1(a_i),\Phi_2(a_i),..,\Phi_k(a_i)]$$

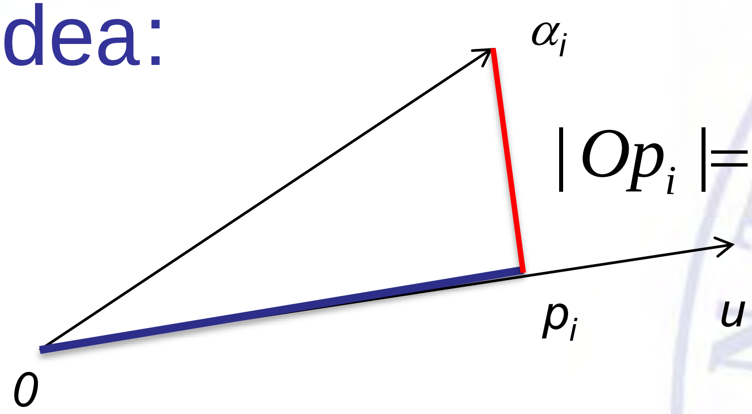
GAIA (2)



Principal component analysis

GAIA (3)

- Idea:



$$|Op_i| = \alpha_i u'$$

$$Min \sum_{i=1}^{n} |\alpha_i p_i|^2 = Max \sum_{i=1}^{n} |Op_i|^2 \Longleftrightarrow Max \;\; uCu'$$

$$u.u' = 1$$

$$\Phi = \begin{pmatrix} \Phi_1(a_i) & \dots & \Phi_k(a_i) \\ \dots & \dots & \dots \\ \Phi_1(a_n) & \dots & \Phi_k(a_n) \end{pmatrix}$$

$$where \;\; nC = \Phi\Phi'$$

GAIA (4)

$$Max\ \ uCu' \qquad u.u'=1$$

$$\Longleftrightarrow$$

$$L(u,\lambda) = uCu' - \lambda.(u.u'-1)$$

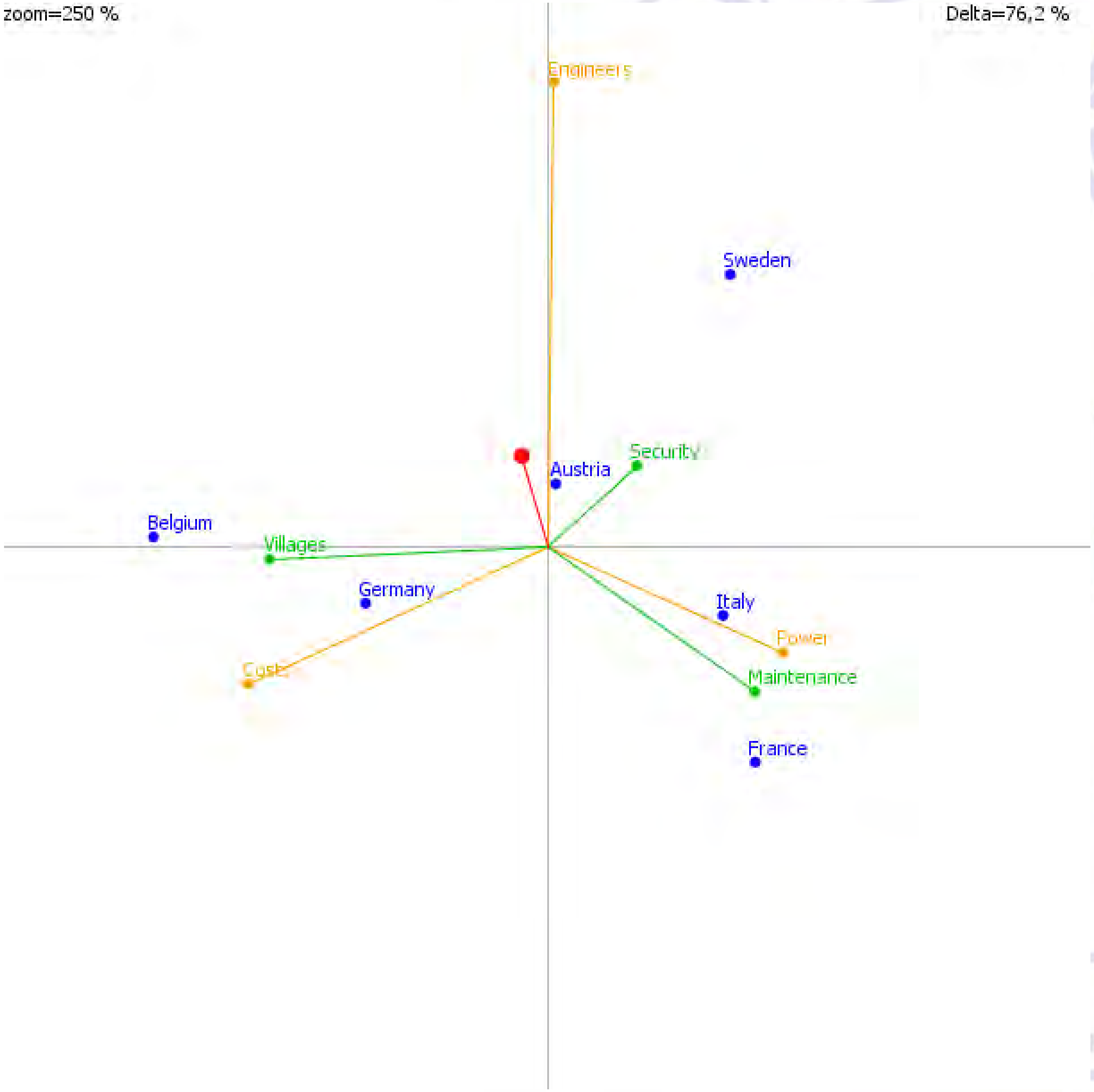
$$\Downarrow$$

$$\begin{cases} C.u = \lambda.u \\ u.u' = 1 \end{cases}$$

$$\Downarrow$$

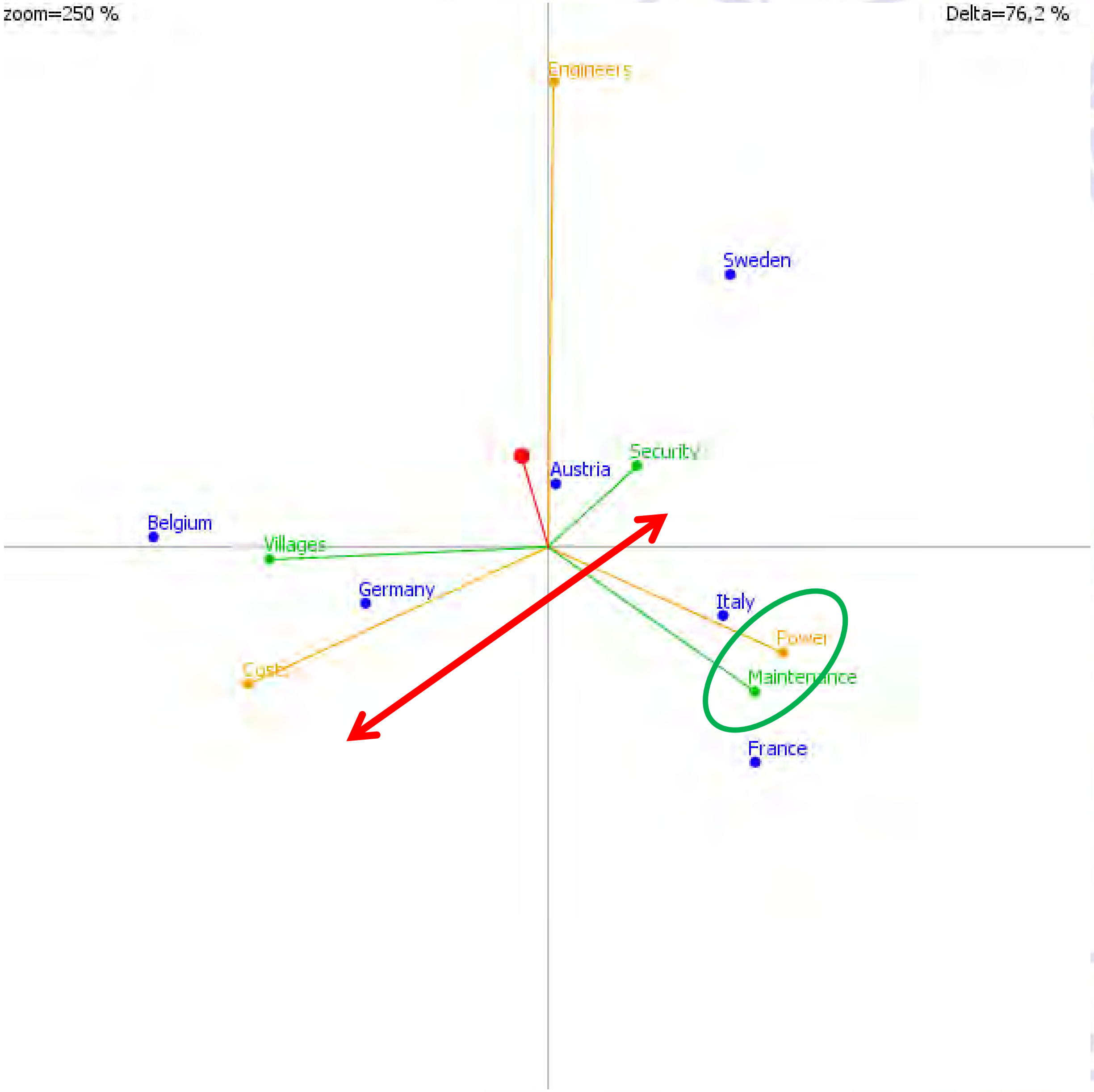
$$Max\ \ uCu' = \lambda_1.u$$

GAIA(3)

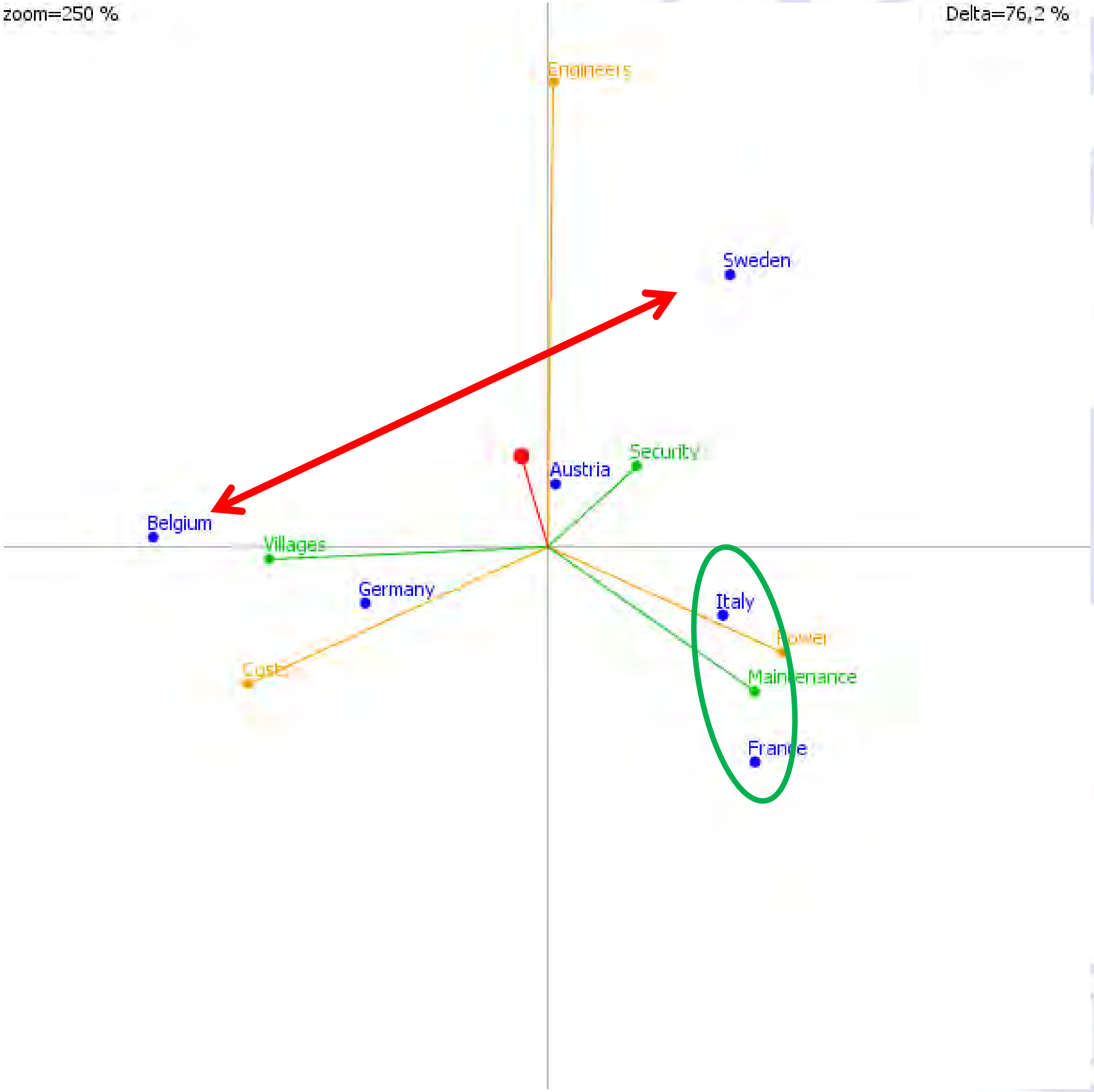


zoom=250 %
Delta=76,2 %
Engineers
Sweden
Security
Austria
Belgium
Villages
Germany
Italy
Power
Cost
Maintenance
France

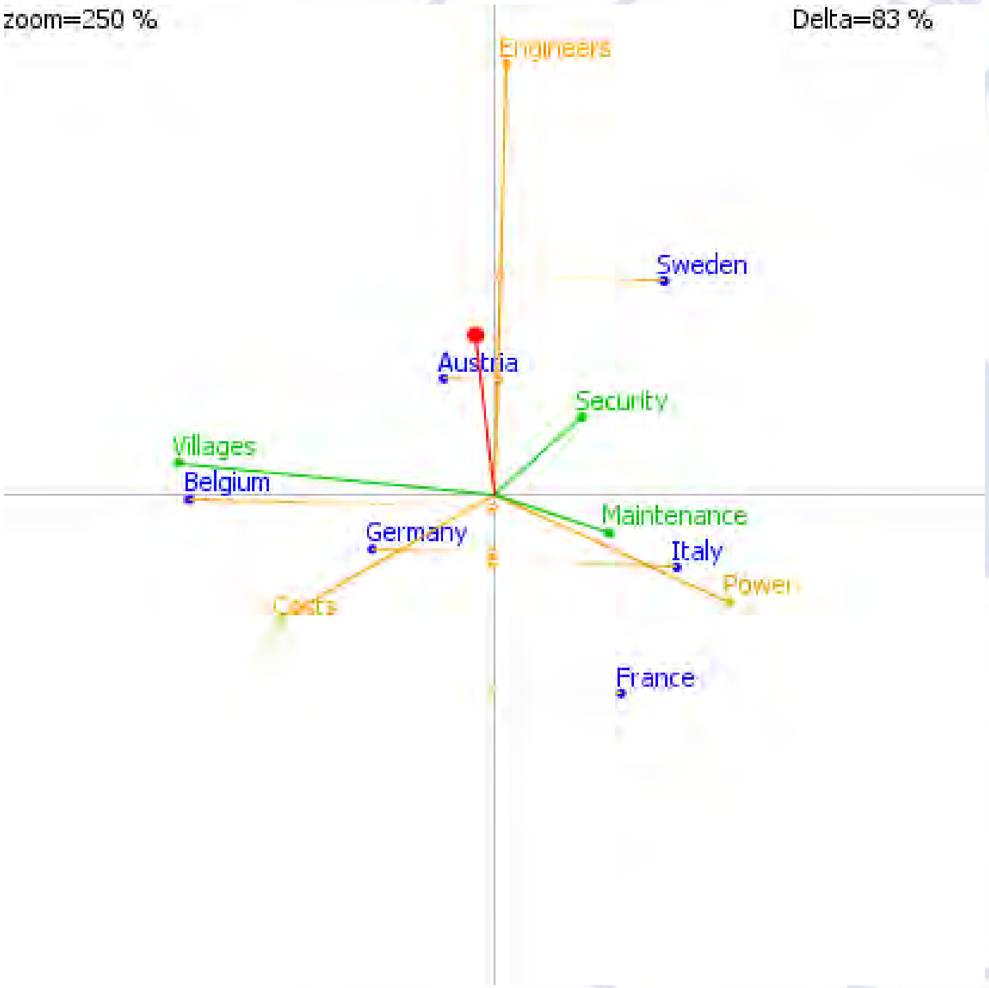
GAIA(4): criteria



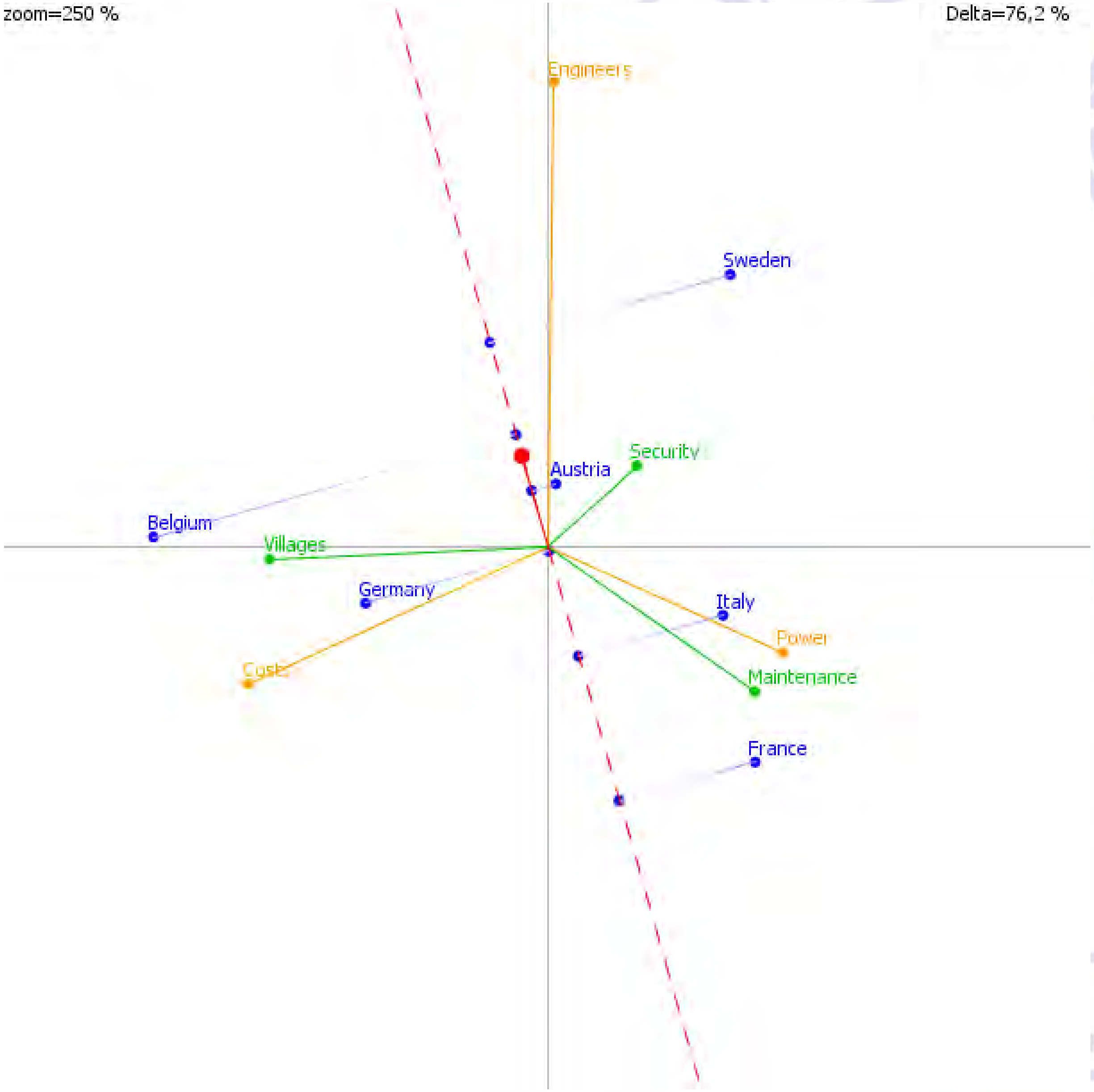
GAIA(5): alternatives



GAIA(6): alternatives / criteria



GAIA(7): Decision stick



Software demonstration

A few words about rank reversal

Rank reversal

- We could have:

$$\pi_{ij} \geq \pi_{ji} \wedge \phi(a_i) \leq \phi(a_j)$$

- In other words: a pairwise rank reversal …
- This opens a discussion about rank reversal …
  - AHP: *Belton and Gear (1983), Saaty and Vargas (1984), Triantaphyllou (2001), Wang and Elhag (2006), Wijnmalen and Wedley (2009)*
  - ELECTRE: *Wang and Triantaphyllou (2005)*
  - PROMETHEE: *De Keyser and Peeters (1996)*
- The concept of rank reversal is not fully formalized (*add a copy of an alternative, deletion of a non discriminating criterion, deletion of an alternative, …)*
- *A direct consequence of Arrow's theorem*

Deletion of a non discriminating criterion

$$\phi_k(a_i) = 0 \forall a_i \in A$$

$$\phi(a_i) = \sum_{j=1}^{q} w_j \phi_j(a_i)$$

$$= \sum_{j=1, j \neq k}^{q} w_j \phi_j(a_i)$$

$$= W_k \sum_{j=1, j \neq k}^{q} \frac{w_j}{W_k} \phi_j(a_i)$$

$$= W_k \sum_{j=1, j \neq k}^{q} w'_j \phi_j(a_i)$$

$$= W_k \phi'(a_i)$$

	f_1	f_2	...	f_k	...	f_q
a_1	$f_1(a_1)$	$f_2(a_1)$	...	α	...	$f_q(a_1)$
a_2	$f_1(a_2)$	$f_2(a_2)$	...	α	...	$f_q(a_2)$
...	...	...	...	...	...	...
a_n	$f_1(a_n)$	$f_2(a_n)$	...	α	...	$f_q(a_n)$

where $W_k = \sum_{j=1, j \neq k}^{q} w_j$ *and* $w'_j = \frac{w_j}{W_k}$

$$\phi(a_i) > \phi(a_j) \Leftrightarrow \phi'(a_i) > \phi'(a_j)$$

Dominance

- Let us assume that:

$$f_k(a_i) \geq f_k(a_j), \forall k = 1,..q$$

- Then:

$$\phi(a_i) = \frac{1}{n-1} \sum_{k=1}^{q} w_k \sum_{b \in A} \pi_k(a_i, b) - \pi_k(b, a_i)$$

$$\geq \frac{1}{n-1} \sum_{k=1}^{q} w_k \sum_{b \in A} \pi_k(a_j, b) - \pi_k(b, a_j) = \phi(a_j)$$

This result holds for any set A such that a_i, $a_j \in A$

More general result (1)

Notations: $A_x = A\backslash\{x\}$, $\Phi_x(a)$

No RR $\Leftrightarrow \big(\Phi(a) - \Phi(b)\big)\big(\Phi_x(a) - \Phi_x(b)\big) > 0$

$$\textit{if} \quad \Phi(a) - \Phi(b) > \frac{[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}$$

No RR (for any action removed) if

$$\Phi(a) - \Phi(b) > \frac{\max_x[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}$$

More general result (2)

➔ *RR can only occur if*

$$\Phi(a) - \Phi(b) < \underbrace{\frac{\max_x[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}}_{\substack{\textit{refined threshold} \\ \textit{(depends on the sample and (a,b))}}} \leq \underbrace{\frac{2}{n-1}}_{\substack{\textit{rough} \\ \textit{threshold} \\ \textit{(constant)}}}$$

Generalization: when k actions are removed

No RR if $\Phi(a) - \Phi(b) > \dfrac{2k}{n-1}$

Mareschal, B., De Smet, Y. and Nemery, P. « *Rank Reversal in the PROMETHEE II Method : Some New Results* », proceedings of de IEEE 2008 International Conference on Industrial Engineering and Engineering Management, Singapore, (2008): 959-963

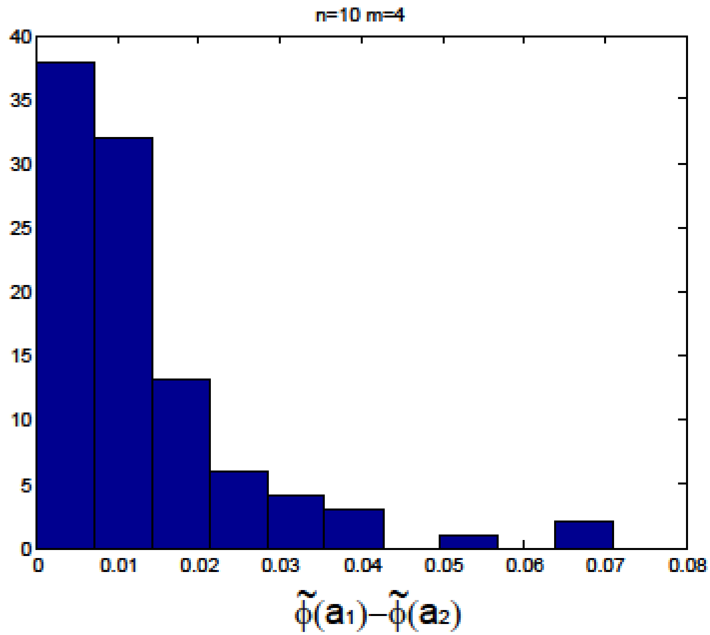
More general result (3)

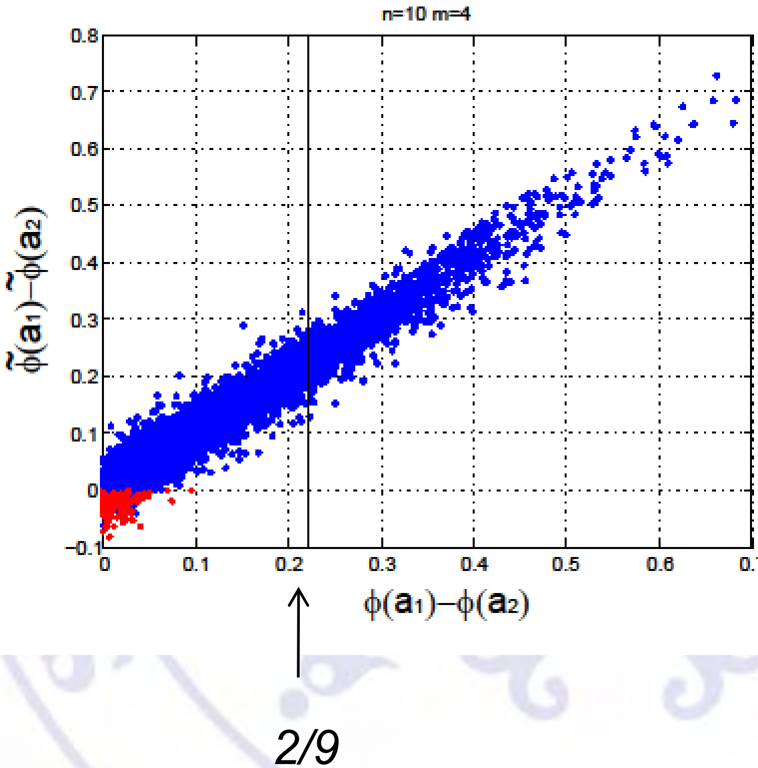
Statistical results relative to the «rough threshold» (for q = 2, DA=Unif)

n	nb RR	$b = \frac{2}{n-1}$	nb $\Delta\Phi \leq b$	nb RR $\mid \Delta\Phi \leq b$
5	2,20 %	0,50	47,4 %	4,6 %
10	0,98 %	0,22	33,5 %	2,9 %
15	0,66 %	0,14	24,7 %	2,6 %
20	0,45 %	0,10	19,9 %	2,2 %
50	0,18 %	0,04	9 %	1,9 %

Conclusion: *The number of RR occurences is really small.*

More general result (4)





Verly, C. and De Smet, Y « Some considerations about rank reversal occurrences in the PROMETHEE methods » to appear in the International Journal of Multicriteria Decision Making.

Related works for PROMETHEE I

- No rank reversal will happen between a_i and a_j if

$$|\phi^{+}(a_i)-\phi^{+}(a_j)|\geq \frac{1}{n-1}$$

$$|\phi^{-}(a_i)-\phi^{-}(a_j)|\geq \frac{1}{n-1}$$

Rank reversal = risk of manipulation

- Joint work with Julien Roland and Céline Verly (to appear in the proceedings of the IPMU 2012 conference)
- Aim: to quantify the likelihood of manipulation in a simplified version of the PROMETHEE II ranking:
  - Usual preference function and equal weights
  - Copeland scores
- More formaly:
  - A given decision maker has a perfect information on the evaluation table;
  - He may propose new alternatives in order to make alternative a_i the first one;
  - Question: how many alternatives are necessary ?

Linear mathematical program

$$\max \quad \sum_{a \in A \cup C} y(a_s, a)$$

$$\begin{aligned}
\text{subject to: } (P_j(a_i, a_j) - 1)\overline{g_k} &< g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C,\ \forall k \in K \\
P_k(a_i, a_j)\overline{g_k} &\geqslant g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C,\ \forall k \in K \\
\pi(a_i, a_j) &= \frac{1}{q} \sum_{k \in K} P_k(a_i, a_j), \forall a_i, a_j \in A \cup C \\
\phi(a) &= \frac{1}{n+m-1} \sum_{x \in A \cup C} \pi(a, x) - \pi(x, a), \forall a \in A \cup C \\
g_k(a) &\leqslant \overline{g_k}, \forall a \in C \\
g_k(a) &\geqslant 0, \forall a \in C \\
2(y(a_s, a) - 1) &\leqslant \phi(a_s) - \phi(a), \forall a \in A \cup C \\
2y(a_s, a) &> \phi(a_s) - \phi(a), \forall a \in A \cup C
\end{aligned}$$

Results for 10 alternatives and 3 criteria

Table 1. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding m well-chosen artificial alternatives.

$j \backslash m$	1	2	3	4	5	6	7	8	9
2	7	3	0	0	0	0	0	0	0
3	37	13	7	0	0	0	0	0	0
4	57	33	17	3	0	0	0	0	0
5	83	63	40	13	0	0	0	0	0
6	90	83	60	23	0	0	0	0	0
7	90	90	77	43	10	0	0	0	0
8	100	100	87	70	37	7	0	0	0
9	100	100	97	83	63	33	3	0	0
10	100	100	97	93	83	63	33	3	0

Comparison with the bound

Table 3. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding m well-chosen artificial alternatives while the Mareschal's bound is not reached.

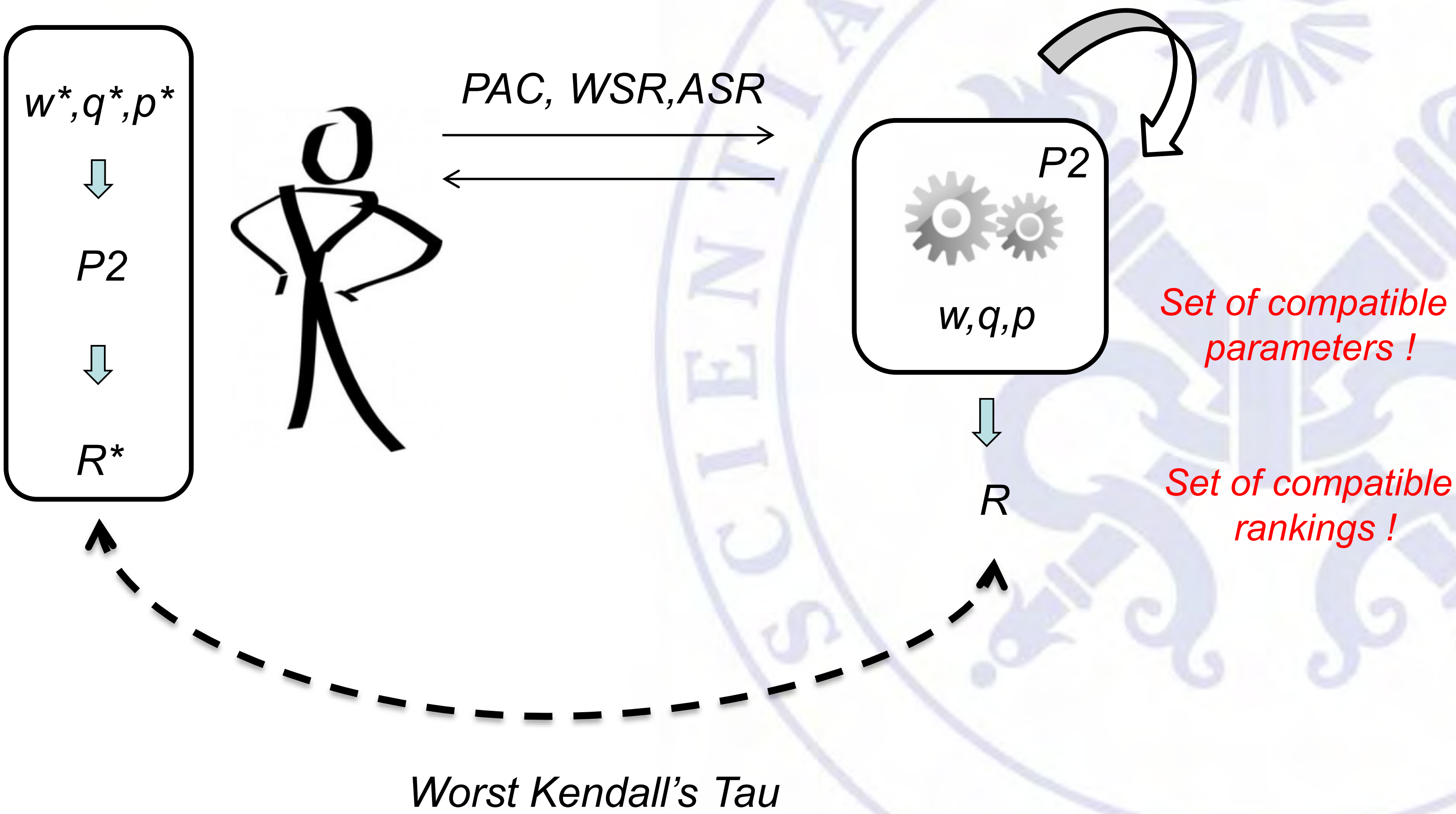
$j\backslash m$	1	2
2	7	3
3	30	13
4	27	33
5	17	63
6	7	83
7	0	90
8	7	73
9	3	40
10	0	20

A few words about preferences elicitation

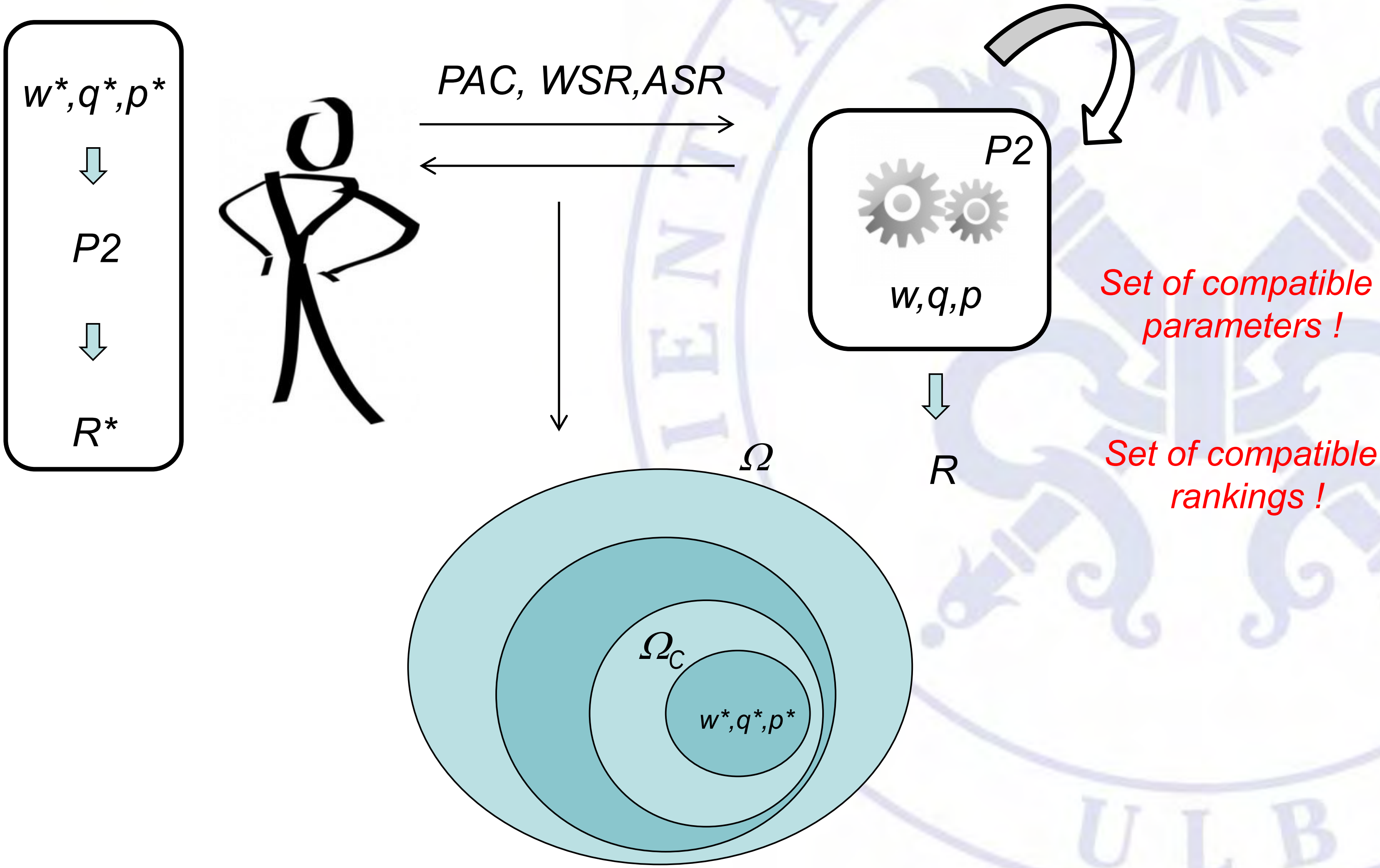
To our knowledge, only a few contributions:

- Özerol, G., Karasakal, E. (2007) « Interactive outranking approaches for multicriteria decision-making problems with imprecise information » JORS, 59, 1253-1268
- Sun, Z. and Han, M. (2010) « Multi-criteria decision making based on PROMETHEE method », in Proceedings of the 2010 international Conference on Computing, Control and Industrial Engineering, 416-418
- Frikha, H., Chabchoub, H., Martel, J.-M. (2010) « *Inferring criteria's relative importance coefficients in PROMETHEE II* », International Journal of Operational Research 7(2), 357-275
- Eppe, S., De Smet, Y., Stützle, T. (2011) « *A bi-objective optimization model to eliciting decision maker's preferences for the PROMETHEE II method* » Proceedings of ADT (2011), 56-66
- Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMTHEE II preference elicitation process: A simulation based approach* » to appear in the proceddings of the IPMU 2012 conference
- Eppe, S., De Smet, Y. « *An experimental parameter space analysis of the PROMETHEE II outranking method* » ongoing work (1)

General idea (1)



General idea (2)



Eppe, S., De Smet, Y., Stützle, T. (2011) « *A bi-objective optimization model to eliciting decision maker's preferences for the PROMETHEE II method* » Proceedings of ADT (2011), 56-66

- Main idea: quality and robustness
- Distinctive feature: the DM may communicate mistakes

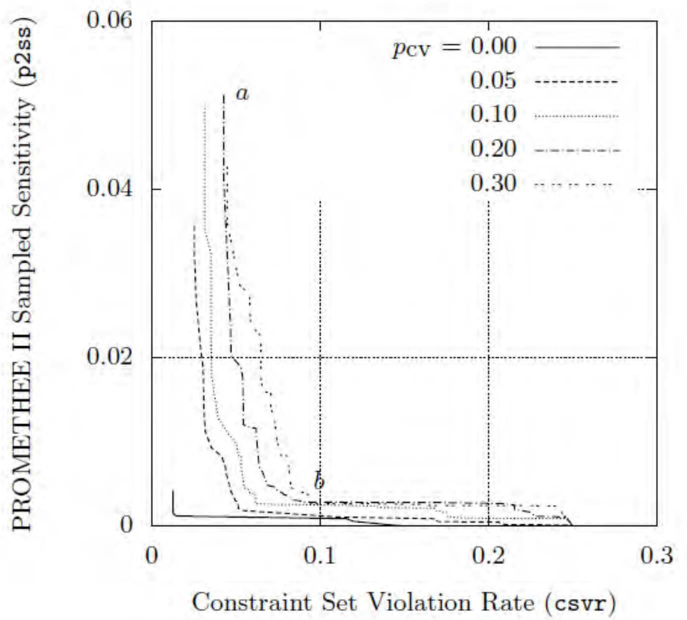


Fig. 4. This plot represents the approximated Pareto frontiers in the objective space, for 20 constraints and several values of the constraint violation rate p_{cv}, i.e., the proportion of inconsistent constraints with respect to the total number of constraints. As expected, increasing the value of p_{cv} has the effect of deteriorating the quality of the solution set both in terms of constraint violation rate and PROMETHEE II sampled sensitivity.

(n=100,q=2)

NSGA-II

Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMETHEE II preference elicitation process: A simulation based approach* » in the proceedings of the IPMU 2012 conference

- Main idea: quantify information infrastructure …

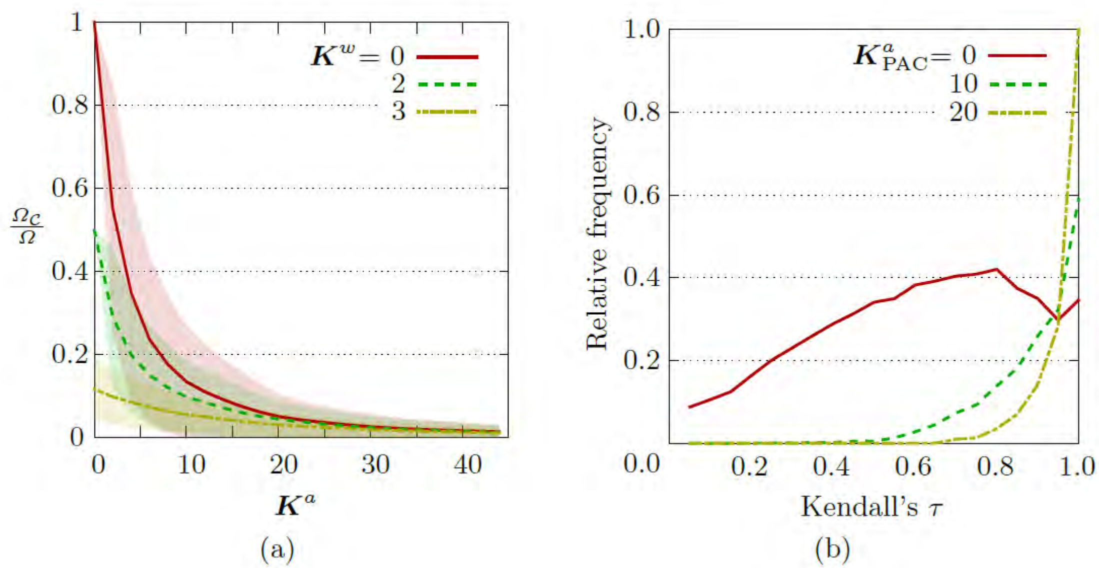


Fig. 3. (a) Evolution of the ratio of compatible weight domain area $\frac{\Omega_C}{\Omega}$ with respect to the whole domain of possible weights Ω, depending on the number of action constraints $\boldsymbol{K}^a$, for different numbers of weight constraints $\boldsymbol{K}^w$. Results are shown for 1000 randomly generated action sets ($n = 10$, $m = 3$) and PAC constraint sets. — **(b)** Distribution of all values of Kendall's τ in the compatible weights domain Ω_C, for respectively $\boldsymbol{K}^a_{\mathrm{PAC}} = 0$, 10, and 20 pairwise action comparisons. No constraints on the weights relative importance are given here ($\boldsymbol{K}^w = 0$).

Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMTHEE II preference elicitation process: A simulation based approach* » to in the proceedings of the IPMU 2012 conference

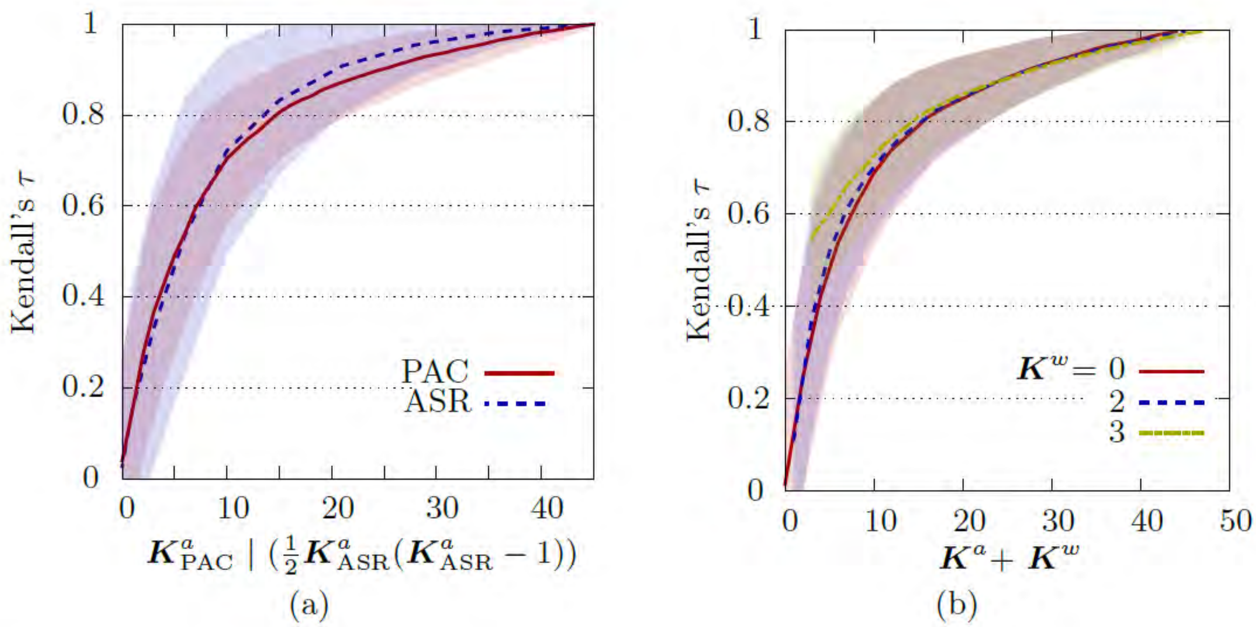


Fig. 4. (a) Evolution of worst Kendall's τ for two different information types: pairwise action comparisons (PAC) and action sub-ranking (ASR). — **(b)** Impact of "weight constraints" on the reachable quality for pairwise action comparisons (PAC). Note that the x-axis represents the sum of action *and* weight constraints, i.e. $\boldsymbol{K}^a + \boldsymbol{K}^w$.

Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMTHEE II preference elicitation process: A simulation based approach* » in the proceddings of the IPMU 2012 conference

Table 1. Number of pairwise action comparisons that have to be given by a DM to reach the desired level of quality $\underline{\boldsymbol{w}}$, assuming that $\boldsymbol{K}^w$ weight constraints have already been provided. The results are shown for randomly generated 3-criteria action sets with a uniform distribution.

	$\underline{\boldsymbol{w}}$						
$\boldsymbol{K}^w$	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0	6	8	10	15	25	34	43
2	4	6	9	14	24	33	43
3	0	1	5	11	22	32	42

Eppe, S., De Smet, Y. « *An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters » accepted for publication in the International of Multicriteria decision making*

- <u>Main idea</u>: to overcome the limitations of the previous approach; pairwise comparisons have to be « well-chosen »
- <u>q-Eval</u>

Table 2 Maximum number of queries that could be generated on average with *q-Eval* for 30 requested queries for 50 randomly generated instances.

	m			
n	3	5	7	10
10	8.2	12.7	15.4	17.4
20	12.3	20.5	26.3	29.5
50	15.88	28.46	30.0	30.0
100	17.26	29.46	30.0	30.0
200	17.96	29.84	30.0	30.0

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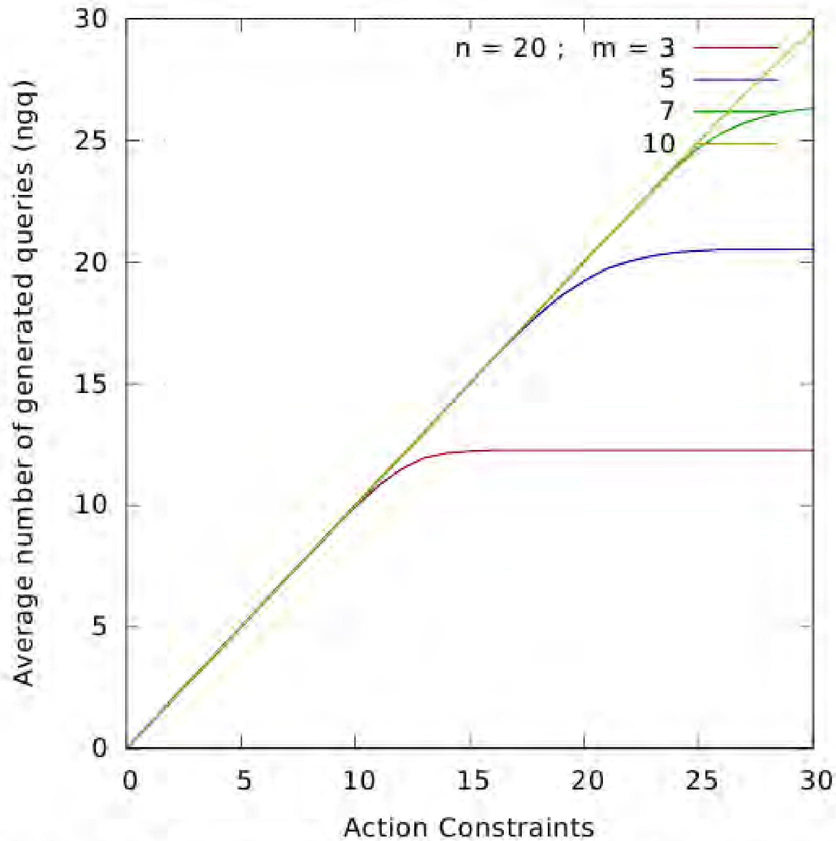

Figure 2 Evolution of the average number of constraints that can be generated with the *q-Eval* method depending on the instance size (number m of alternatives) for a given number of alternatives. Plots for a constant value of m and different number of alternatives n are very similar.

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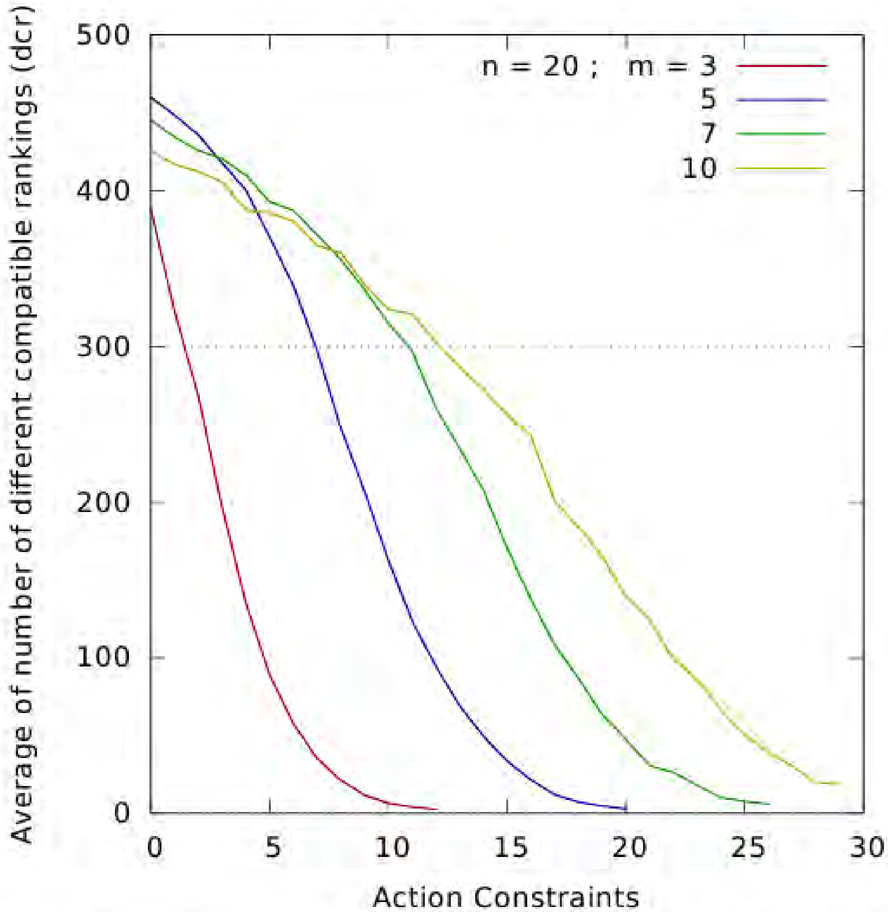

Figure 3 For $n = 20$ actions, shows the average evolution of the number of different rankings found in the sampling of the compatible parameter domain Ω, for different numbers m of criteria.

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Table 3 This table represents the average value of the worst Kendall's correlation coefficient τ_G for the sampling of the compatible parameter domain Ω for each trial after the last query. Note that the sample size of $n_S = 500$ has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

	m			
n	3	5	7	10
10	0.999	0.998	0.988	0.988
20	1.000	1.000	0.999	0.994
50	1.000	1.000	0.998	0.987
100	1.000	1.000	0.997	0.983
200	1.000	1.000	0.998	0.982

Table 4 This table represents the average number of different rankings in the samples for each trial after the last query. Note that the sample size of $n_S = 500$ has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

	m			
n	3	5	7	10
10	1.1	1.1	2.4	3.9
20	1.0	1.5	2.2	14.1
50	4.8	18.6	204.4	358.8
100	44.6	226.7	430.8	465.9
200	187.9	432.1	477.8	490.5

Eppe, S., De Smet, Y. « *An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters » accepted for publication in the International of Multicriteria decision making*

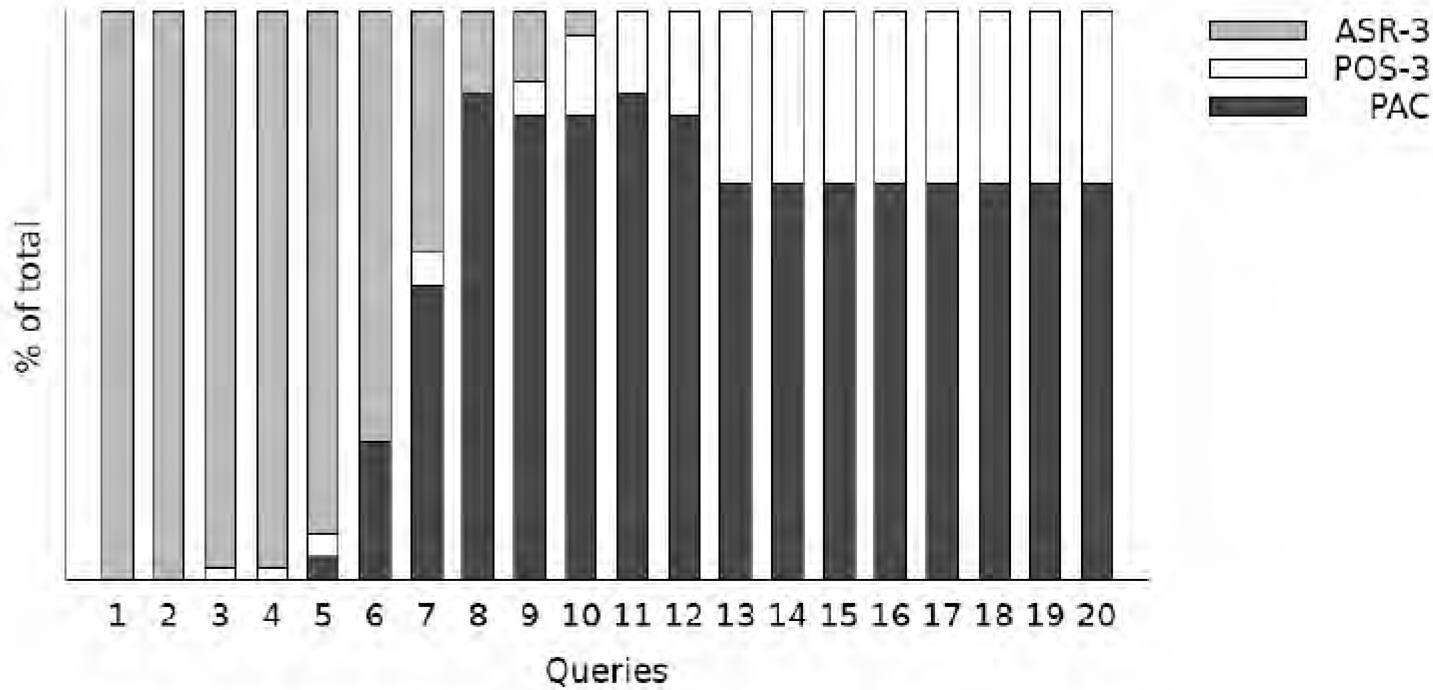
Table 12 Number of queries that has to be answered by a DM to reach the desired level of quality $\underline{\tau_G}$ for randomly generated action sets (n = 20 actions and m = 5 criteria)

		$\underline{\tau_G}$						
		0.50	*0.60*	*0.70*	*0.80*	*0.90*	*0.95*	*0.99*
Random	PAC	13	17	21	25	35	47	
Q-Eval	PAC	4	5	6	7	9	11	17
	POS-3	2	3	4	4	6	8	
	ASR-3	2	3	4	4	4	5	
	Adaptive	2	2	3	3	4	5	10

Note: The extended Q-Eval method significantly outperforms the bottom-line approach (Eppe and De Smet, 2012), presented in the first row, and the original *Q-Eval*, shown in the second row.

Eppe, S., De Smet, Y. « *An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters* » *accepted for publication in the International of Multicriteria decision making*

Figure 9 Evolution of the mean ratio of each *Q-Eval* query-type when applying the *adaptive query selection scheme*



Note: Each type of query plays a role and has its usefulness in the process at different stages of the eliciting process.

Directions for future research

1. Theoretical foundations of the PROMETHEE & GAIA methods
   - Rank reversal
2. Preferences' elicitation
3. Current directions
   - GIS, DEA, Uncertainty (see Mareschal et al.), Sorting (see Nemery et al.), …