UTA METHODS & APPLICATIONS
PART I

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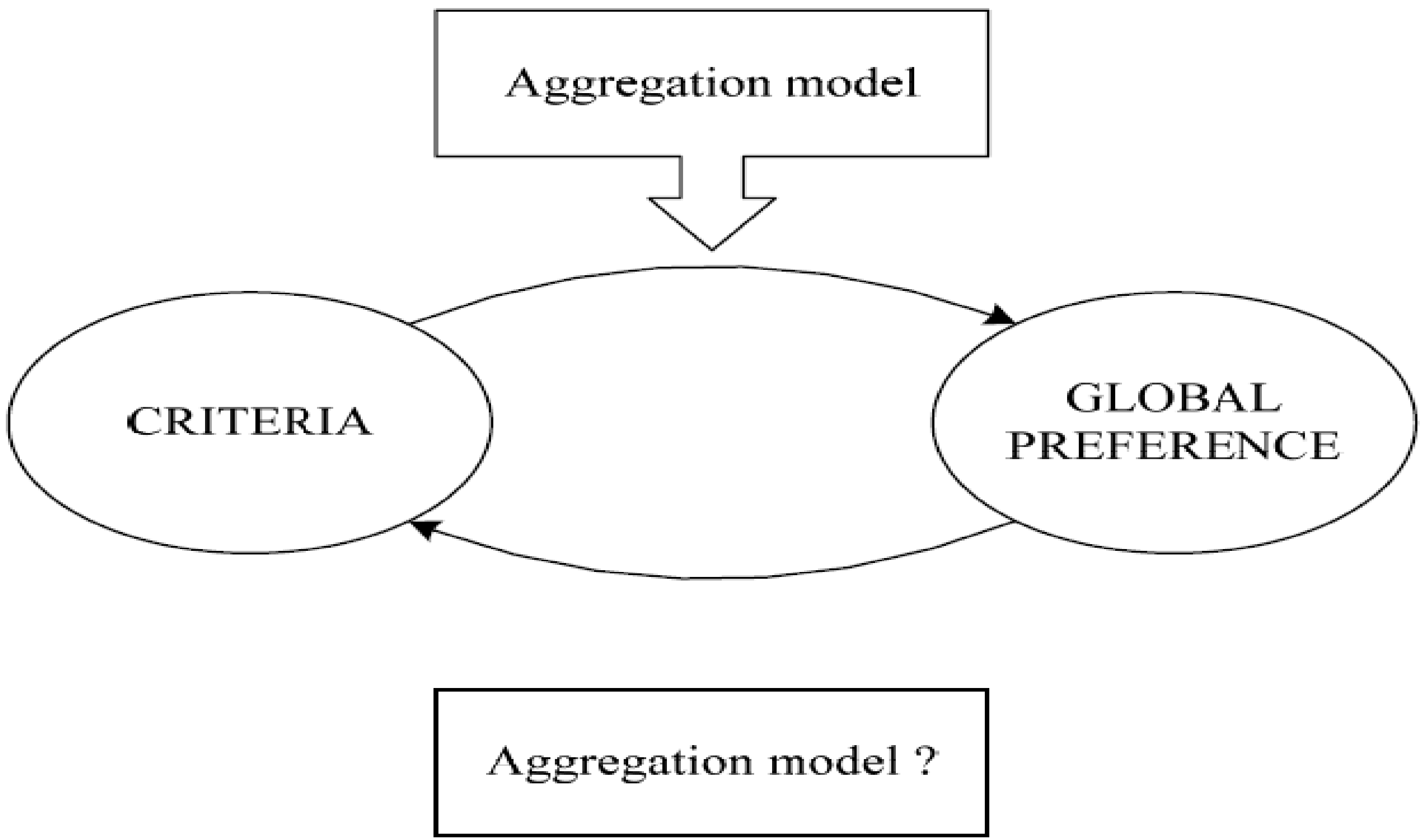
THIS PRESENTATION
IS DEDICATED
TO THE MEMORY

OF

ERIC JACQUET-LAGRÈZE
(1947–2017)

THE AGGREGATION AND DISAGGREGATION PARADIGMS IN MCDA

(JACQUET-LAGRÈZE AND SISKOS, 1978, 2001)



OUTLINE

- Introduction
- Multicriteria modeling & notation
- Aggregation–disaggregation philosophy
- The famous UTA methods
- A numerical example
- UTA based decision support systems

THE MULTICRITERIA ANALYSIS PHILOSOPHY

1. A set of methods or models enabling the aggregation of multiple evaluation criteria to choose one or more actions from a set A.

2. An activity of decision-aid to a well-defined decision-maker.

A GENERAL MODELING METHODOLOGY (Roy, 1985)

- ***Level 1***: Object of the decision, including the definition of the set of potential actions A and the determination of a problematic on A.
- ***Level 2***: Modeling of a consistent family of criteria assuming that these criteria are non-decreasing value functions, exhaustive and non-redundant.
- ***Level 3***: Development of a global preference model, to aggregate the marginal preferences on the criteria.
- ***Level 4***: Decision-aid or decision support, based on the results of level 3 and the problematic of level 1.

THE FOUR PROBLEMATICS IN MCDA

- ***Problematic α***: Choosing one action from A (choice).
- ***Problematic β***: Sorting the actions into pre-defined and preference-ordered categories (sorting).
- ***Problematic γ***: Ranking the actions from the best one to the worst one (ranking).
- ***Problematic δ***: Describing the actions in terms of their performances on the criteria (description).

CRITERIA MODELING PROCESS

Each criterion is a non-decreasing real valued function defined on A:

$$g_i : A \rightarrow \left[g_{i*}, g_i^* \right] \subset \Re \,/\, a \rightarrow g(a) \in \Re$$

where:

$\left[g_{i*}, g_i^* \right]$: the criterion evaluation scale

g_{i*} : the worst level of the i - th criterion

g_i^* : the best level of the i - th criterion

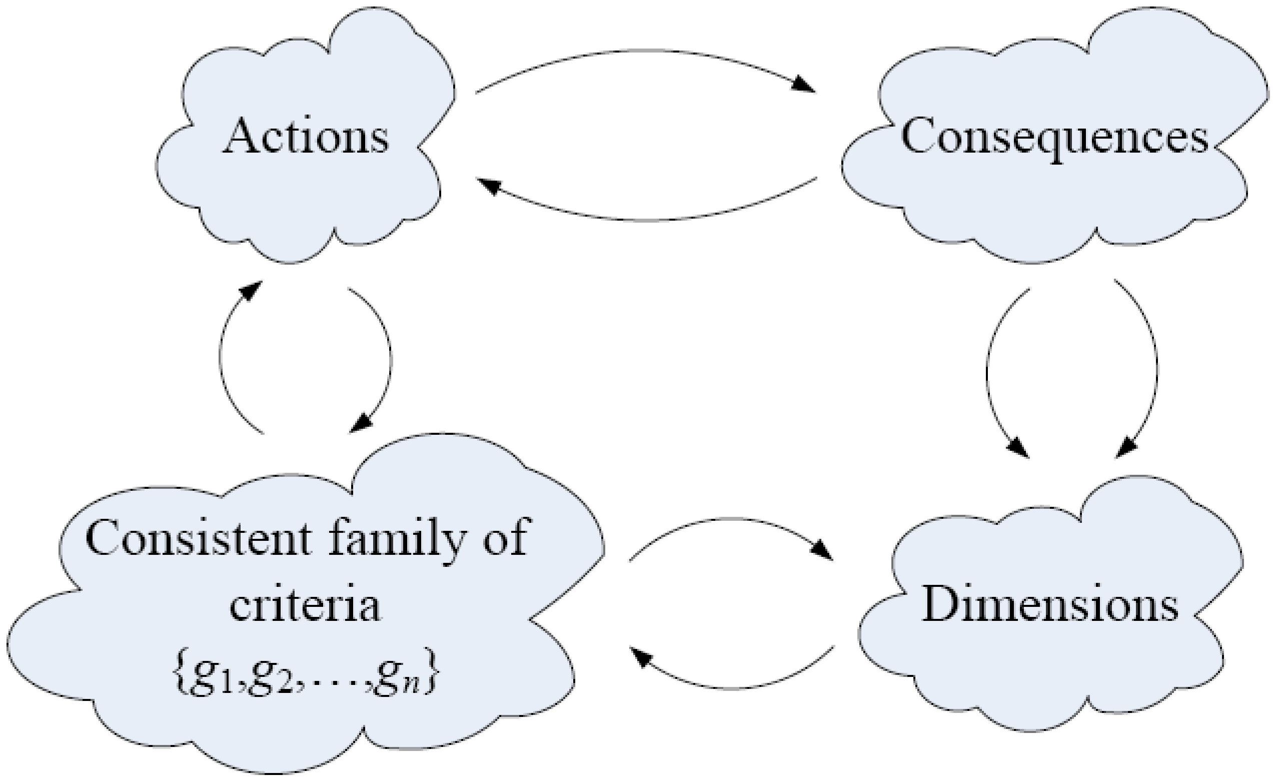
$g_i(a)$: the evaluation or performance of action a on the i - th criterion

$\boldsymbol{g}(a)$: the vector of n performances $\left[g_1(a), g_2(a), \ldots, g_n(a) \right]$

From the above definitions the following preferential situations can be determined:

$$\begin{cases} g_i(a) > g_i(b) \Leftrightarrow a \succ b \left(a \text{ is prefered to } b \right) \\ g_i(a) = g_i(b) \Leftrightarrow a \sim b \left(a \text{ is indifferent to } b \right) \end{cases}$$

ANALYSIS OF CONSEQUENCES AND ELABORATION OF CRITERIA #1



ANALYSIS OF CONSEQUENCES AND ELABORATION OF CRITERIA #2

- Consistent family of criteria: $\{g_1, g_2, \ldots, g_n\}, g_i : A \rightarrow \left[g_{i*}, g_i^*\right] \subset \Re$

1. **Completeness**

$g_i(a) = g_i(a') \; \forall i = 1,2,\ldots n \Rightarrow a\mathrm{I}a'$ (indifference)

2. **Monotonicity**

$g_i(a) = g_i(a') \; \forall i \neq k$ and $g_k(a) > g_k(a') \Rightarrow a\mathrm{P}a'$ (*preference*)

3. **Non redundancy**

If the elimination of any single criterion from a criteria set that satisfies the monotonicity and completeness conditions leads to the formation of a new criteria set that does not meet these conditions, then the initial set of criteria is considered to be non-redundant.

School's Case Study
Urban Sustainability Assessment I

Set of Actions

A = {Beijing, Berlin, Copenhagen, Hong Kong, London, New York, Paris, Prague, Seoul, Shanghai, Stockholm, Tokyo, Warsaw}

Problematic

Provide an evaluation result that has one of the three alternative forms:

- A global score for each city measuring the value of sustainability of the city.

Suggested methods: **ROBUST UTA Method, MAVT Methodology, PROMETHEE II Method & Robust Simos Method, AHP or MACBETH Method**

- A complete ranking of the cities (**γ**)

Suggested methods: **ELECTRE III Method**

- An assignment of each city to one of the four ordered sustainability categories (**β**):

Green category (Very strong sustainability)

Blue category (Strong sustainability)

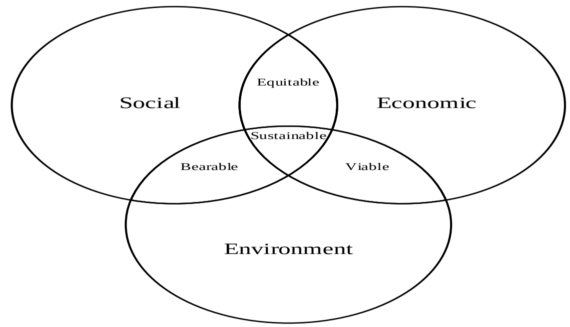
Yellow category (Moderate sustainability)

Red category (Poor sustainability)

Suggested methods: **ELECTRE TRI Method, UTADIS Method**

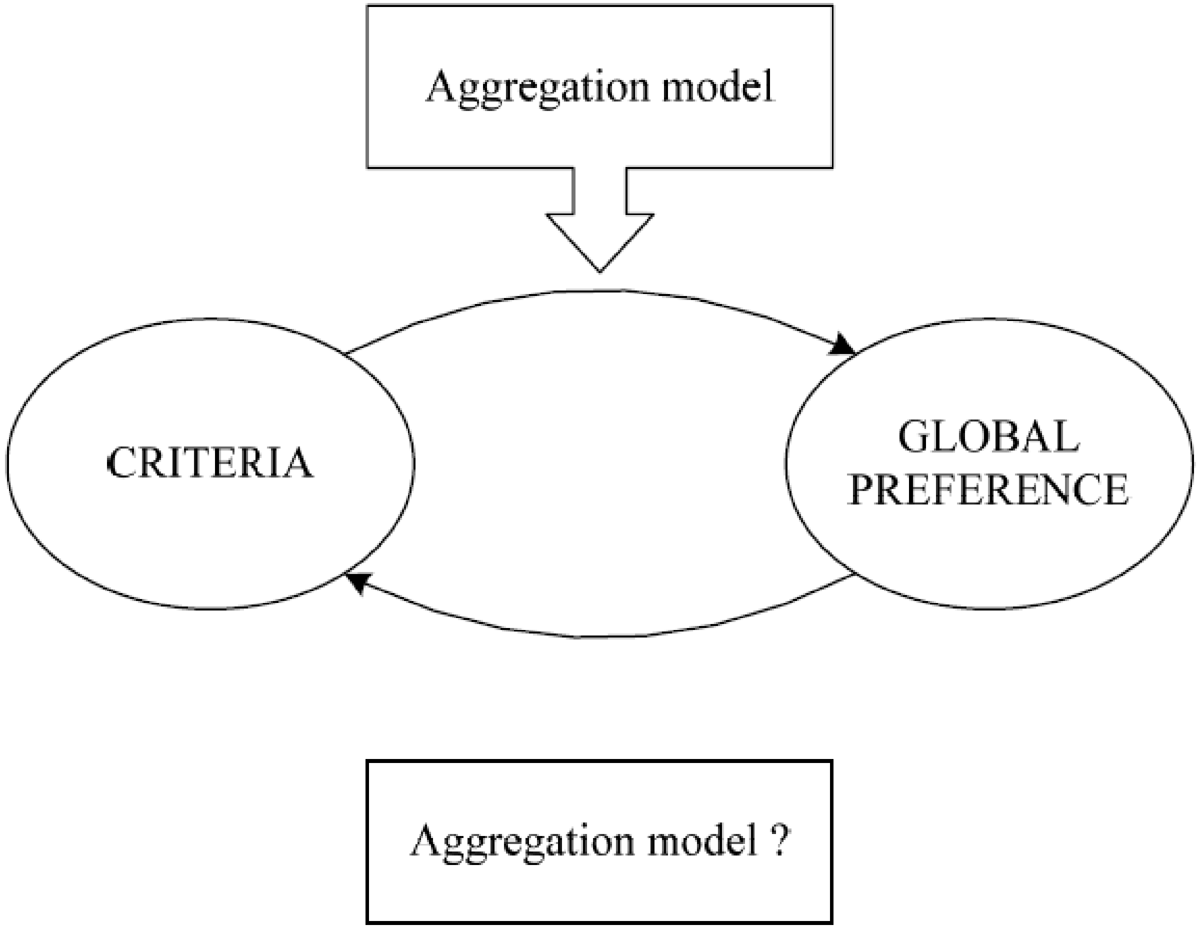
School's Case Study
Urban Sustainability Assessment II

Set of Dimensions

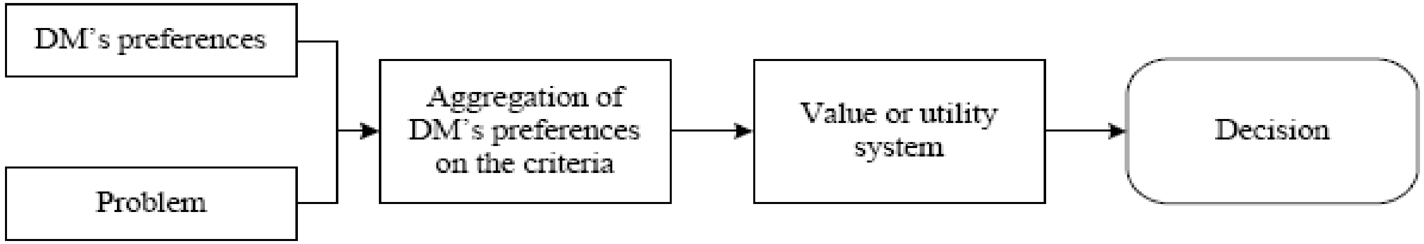


- Employment
- Doctor resource
- Education
- Pensions
- Healthcare
- Air pollution
- Industrial pollution
- Air qualified days
- Wastewater treatment
- Household waste management
- Urban density
- Mass transit usage
- Public green space
- Public water supply
- Internet access
- Income level
- Reliance on heavy industry
- Capacity investment
- Energy consumption
- Power efficiency
- Water efficiency

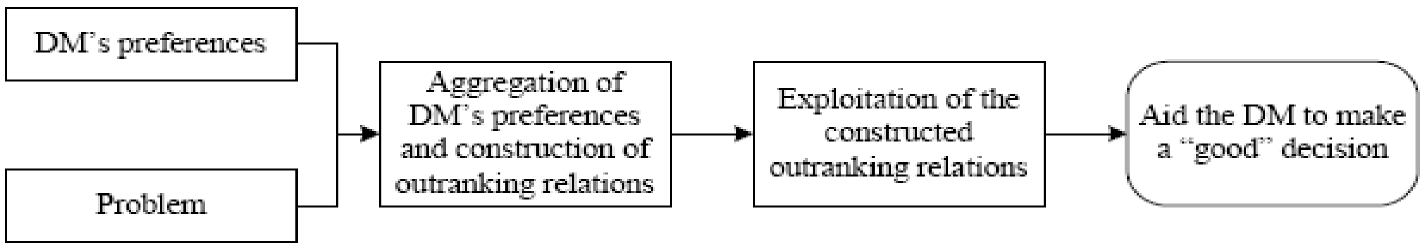
THE AGGREGATION AND DISAGGREGATION PARADIGMS IN MCDA
(JACQUET-LAGRÈZE AND SISKOS, 1978, 2001)



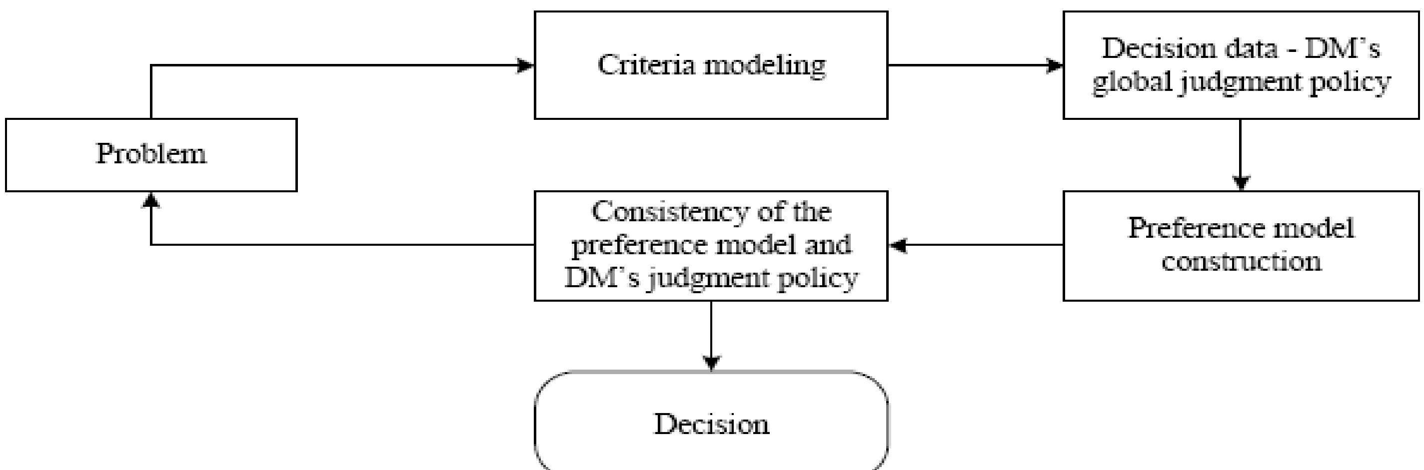
DISAGGREGATION-AGGREGATION APPROACH VS. OTHER MCDA APPROACHES (SISKOS AND SPYRIDAKOS, 1999) #1



(a) The value system approach

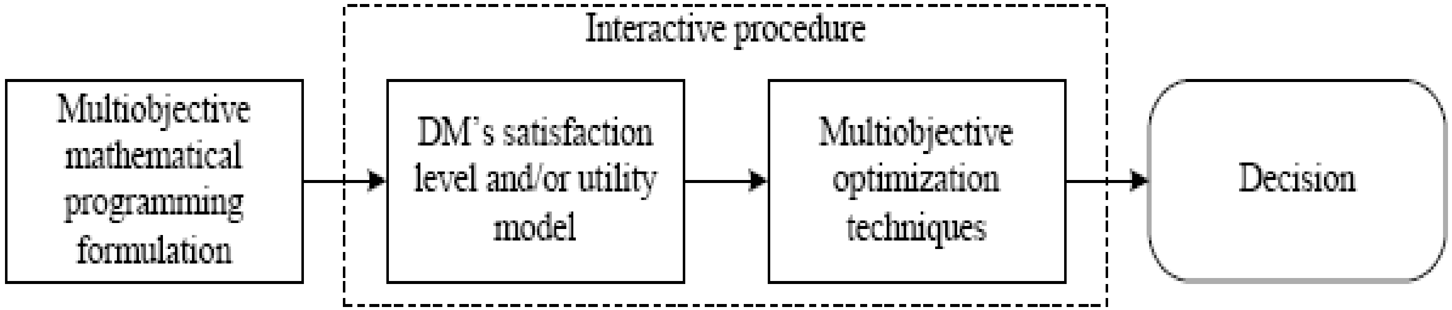


(b) The outranking relation approach



(c) The disaggregation-aggregation approach

DISAGGREGATION-AGGREGATION APPROACH VS. OTHER MCDA APPROACHES (SISKOS AND SPYRIDAKOS, 1999) #2



(d) The multiobjective optimization approach

THE UTA METHOD(S) #1

Principle

- The UTA (UTilités Additives) method proposed by Jacquet-Lagrèze and Siskos (1982) aims at **inferring one or more additive value functions** from a given ranking on a reference set A_R.

- The method uses special linear programming techniques to assess these functions so that the ranking(s) obtained through these functions on A_R is (are) as consistent as possible with the given one.

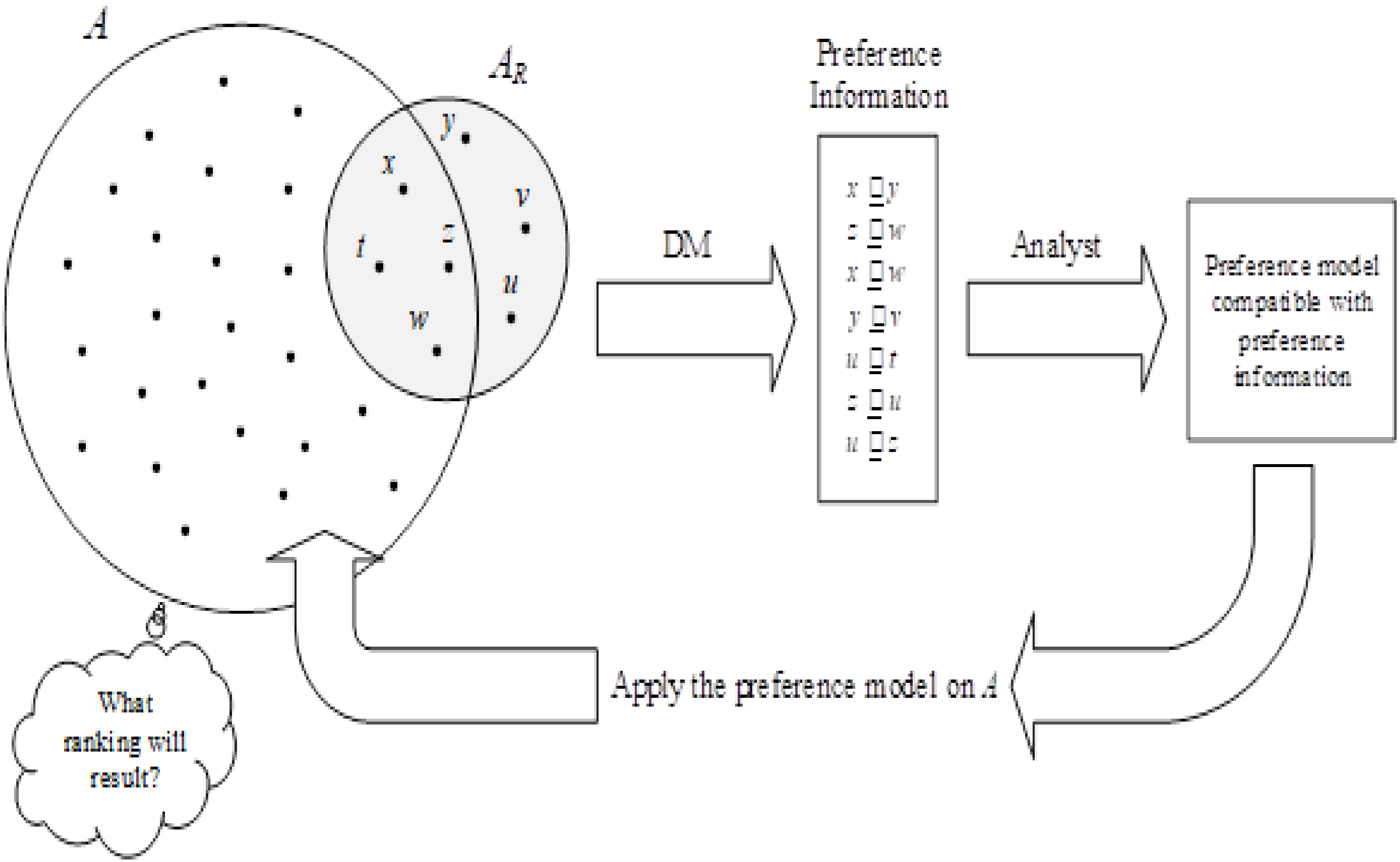
REFERENCE ACTIONS A_R

The clarification of the DM's global preference necessitates the use of a set of reference actions A_R. Usually, this set could be:

1. A set of past decision alternatives (A_R : past actions)
2. A subset of actions, especially when A is large $\left(A_R \subset A\right)$
3. A set of fictitious actions, consisting of performances on the criteria, which can be easily judged by the decision-maker to perform global comparisons (A_R : fictitious actions)

In each of the above cases, the DM is asked to externalize and/or confirm his/her global preferences on the set A_R taking into account the performances of the reference actions on all criteria.

THE PREFERENCE DISAGGREGATION PROCEDURE



THE UTA METHOD(S) #2

The additive value model

- The criteria aggregation model in UTA is assumed to be an additive value function of the following form:

$$u(\boldsymbol{g}) = \sum_{i=1}^{n} p_i u_i(g_i)$$

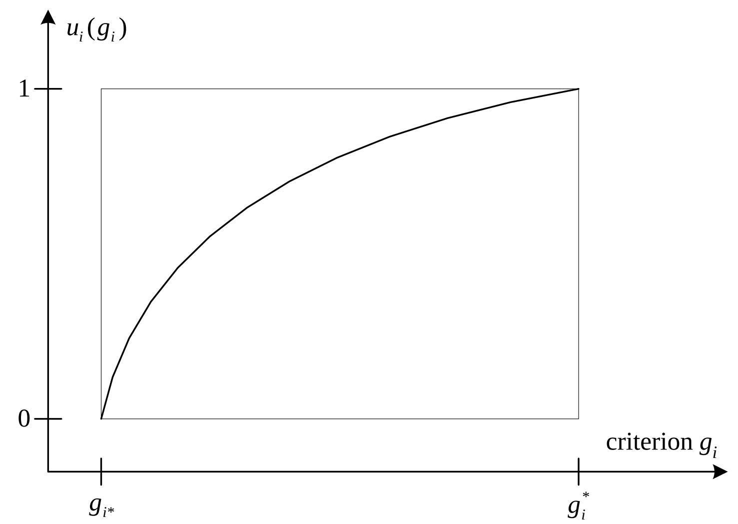
subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} p_i = 1 \\ u_i(g_{i*}) = 0, u_i(g_i^*) = 1 \; \forall \; i = 1,2,\ldots,n \end{cases}$$

where $u_i, i = 1,2,\ldots,n$, are non-decreasing and **piecewise-linear** real valued functions, named marginal value functions, which are normalized between 0 and 1, and p_i is the weight of u_i.

THE UTA METHOD(S) #3

The additive value model



Normalized value function

THE UTA METHOD(S) #4

The additive value model

- Both the marginal and the global value functions have the monotonicity property of the true criterion. For instance, in the case of the global value function the following properties hold:

$$\begin{cases} u[\boldsymbol{g}(a)] > u[\boldsymbol{g}(b)] \Leftrightarrow a \succ b \, (preference) \\ u[\boldsymbol{g}(a)] = u[\boldsymbol{g}(b)] \Leftrightarrow a \sim b \, (indifference) \end{cases}$$

- The UTA method infers an unweighted equivalent form of the additive value function:

$$u(\boldsymbol{g}) = \sum_{i=1}^{n} u_i(g_i)$$

THE UTA METHOD(S) #5

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} u_i\left(g_i^*\right) = 1 \\ u_i\left(g_{i*}\right) = 0 \ \forall\ i = 1,2,\ldots,n \end{cases}$$

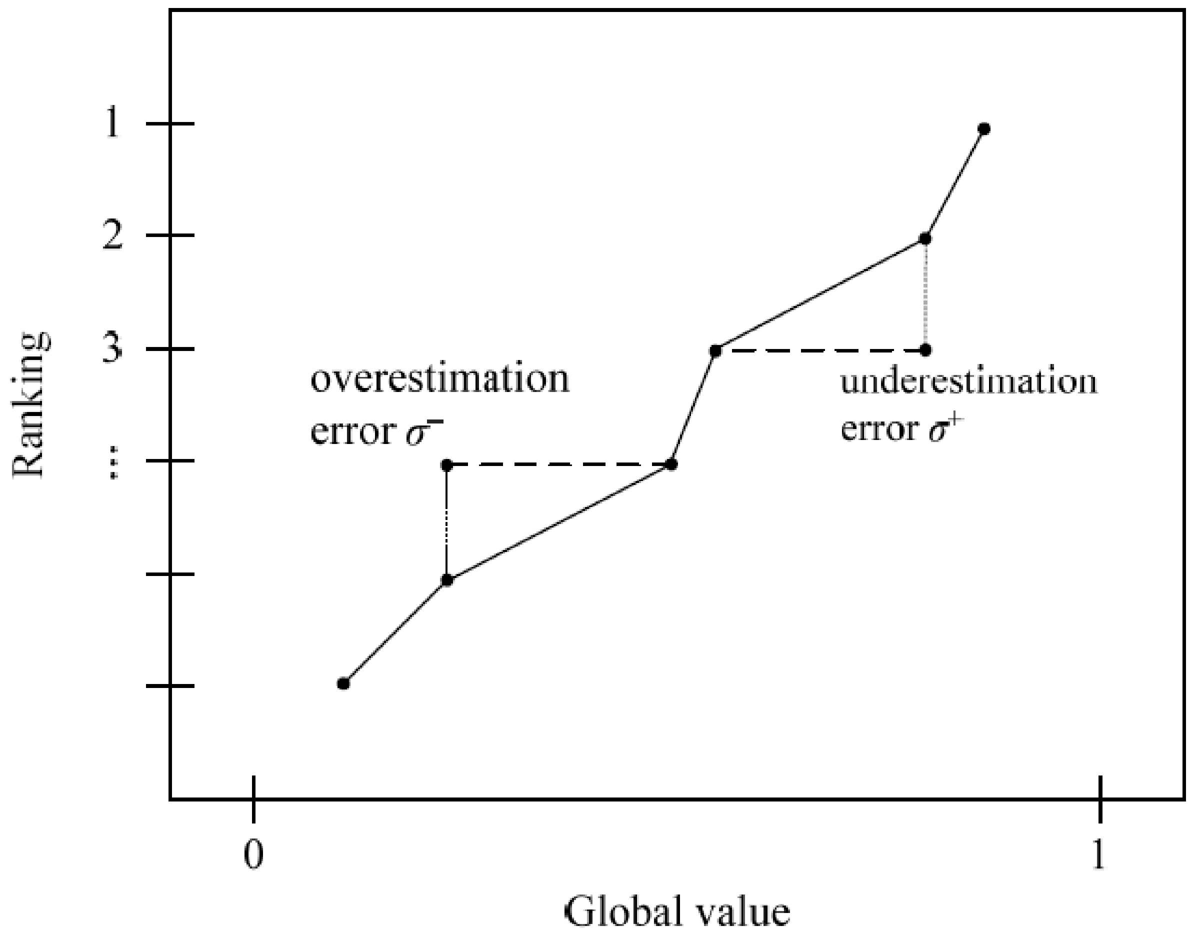
Of course, the existence of such a preference model assumes the preferential independence of the criteria for the DM (Keeney and Raiffa, 1976).

THE UTASTAR ALGORITHM #1

Principle

- The UTASTAR method proposed by Siskos and Yannacopoulos (1985) is an improved version of the original UTA method.
- In the original version of UTA (Jacquet-Lagrèze and Siskos, 1982), for each reference action $a \in A_R$, *a* single error $\sigma(a)$ is introduced to be minimized. This error function is not sufficient to minimize completely the dispersion of points all around the regression curve. The problem is posed by points situated on the right of the curve, from which it would be suitable to subtract an amount of value/utility and not increase the values/utilities of the others.

THE UTASTAR ALGORITHM #2



Ordinal regression curve (ranking versus global value)

THE UTASTAR ALGORITHM #3

Initialization

- In UTASTAR method, Siskos and Yannacopoulos (1985) introduced a double positive error function:

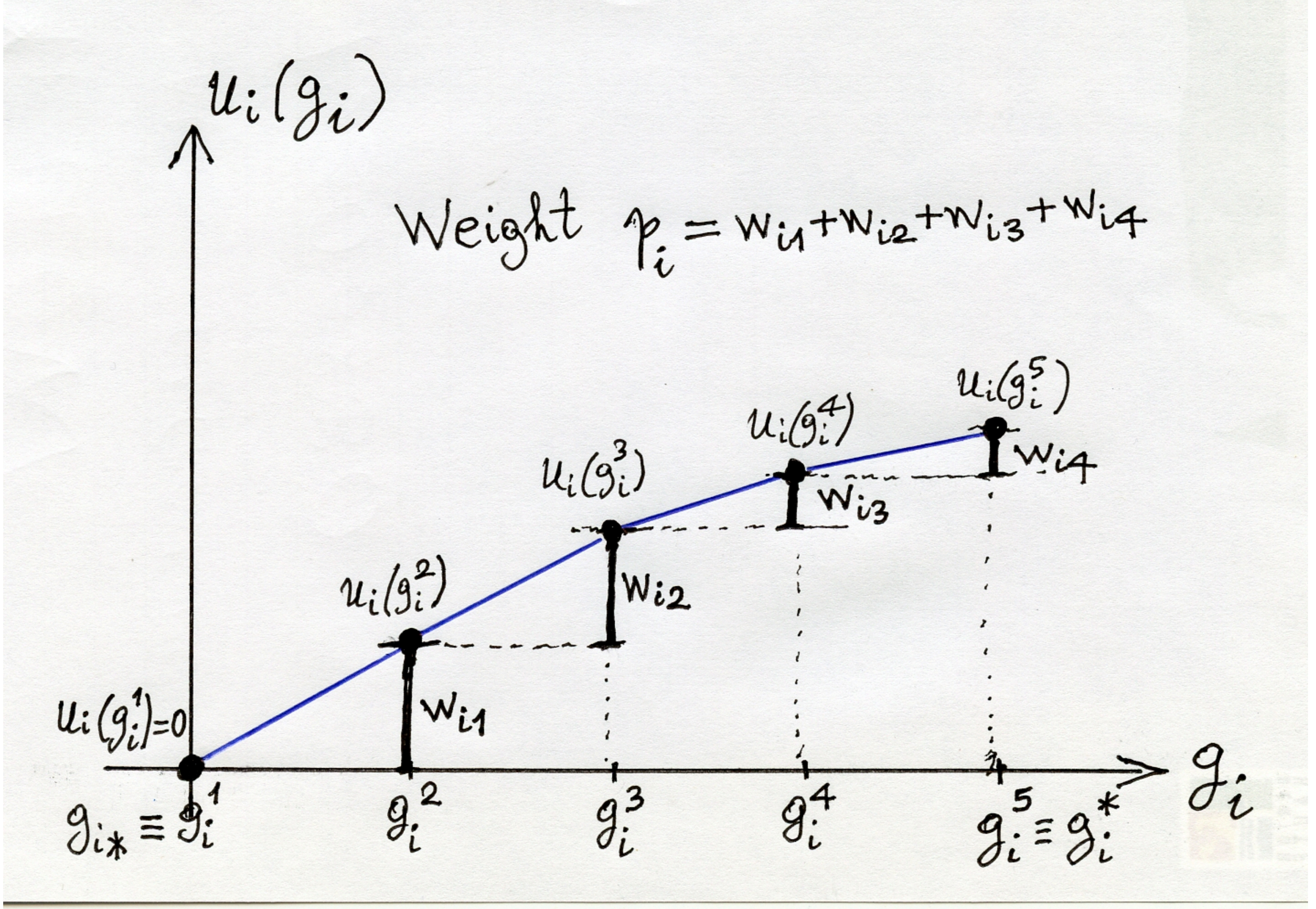
$$u'[\boldsymbol{g}(a)] = \sum_{i=1}^{n} u_i[g_i(a)] - \sigma^+(a) + \sigma^-(a) \; \forall \, a \in A_{\mathfrak{R}}$$

  where σ^+ and σ^- are the overestimation and the underestimation error respectively.

- For each criterion *i* the evaluation scale is discretized into α_i levels, and the monotonicity constraints of the criteria are taken into account through the transformations:

$$w_{ij} = u_i\left(g_i^{j+1}\right) - u_i\left(g_i^{j}\right) \geq 0 \; \forall \, i = 1,2,...,n \text{ and } j = 1,2,...,a_i - 1$$

THE UTA UKNOWN VARIABLES



THE UTASTAR ALGORITHM #4

The algorithm

Step 1:

- Express the global value of reference actions $u[\mathbf{g}(a_k)]$, $k=1,2,\ldots,n$, first in terms of piecewise-linear marginal values $u_i(a_i)$, and then in terms of variables w_{ij}, by means of the following expressions:

$$\begin{cases} u_i\left(g_i^1\right)=0 \ \forall\, i=1,2,\ldots,n \\ u_i\left(g_i^j\right)=\sum_{t=1}^{j-1} w_{ij} \ \forall\, i=1,2,\ldots,n \text{ and } j=2,3,\ldots,a_i-1 \end{cases}$$

THE UTASTAR ALGORITHM #5

The algorithm

Step 2:

- Introduce two error functions σ^+ and σ^- on A_R by writing for each pair of consecutive actions in the ranking the analytic expressions:

$$\Delta(a_k, a_{k+1}) = u[\mathbf{g}(a_k)] - \sigma^+(a_k) + \sigma^-(a_k) - u[\mathbf{g}(a_{k+1})] + \sigma^+(a_{k+1}) - \sigma^-(a_{k+1})$$

THE UTASTAR ALGORITHM #6

The algorithm

Step 3:

- Solve the linear program:

$$\begin{cases} [min]z = \sum_{k=1}^{m}\left[\sigma^{+}(a_k)+\sigma^{-}(a_k)\right] \\ \text{subject to} \\ \left.\begin{array}{l} \Delta\left(a_k, a_{k+1}\right) \geq \delta \text{ if } a_k \succ a_{k+1} \\ \Delta\left(a_k, a_{k+1}\right) = 0 \text{ if } a_k \sim a_{k+1} \end{array}\right\} \forall k \\ \sum_{i=1}^{n}\sum_{j=1}^{a_i-1} w_{ij} = 1 \\ w_{ij} \geq 0, \sigma^{+}(a_k) \geq 0, \sigma^{-}(a_k) \geq 0 \; \forall i, j \text{ and } k \end{cases}$$

with δ a small positive number

THE UTASTAR ALGORITHM #7

The algorithm

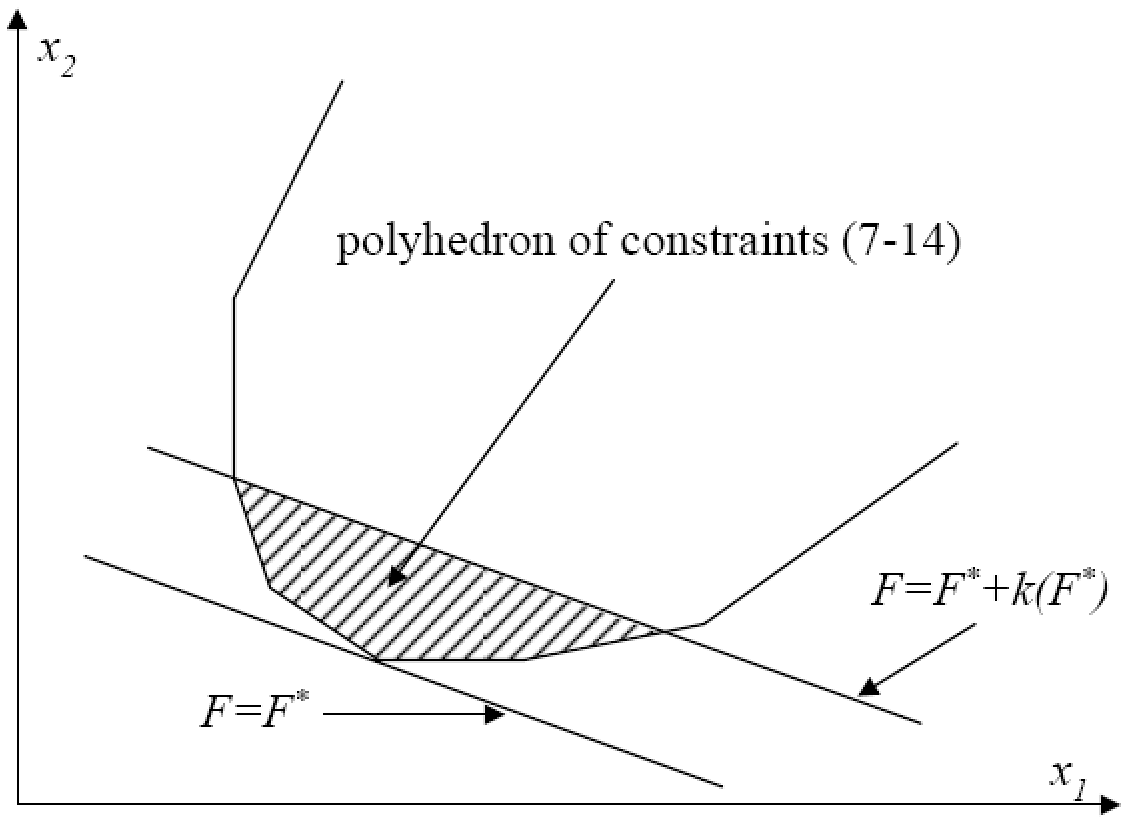
Step 4:

Test the existence of multiple or near optimal solutions of the linear program (stability/robustness analysis); in case of non uniqueness, find the mean additive value function of those (near) optimal solutions which maximize the objective functions:

$$\sum_{k=1}^{m}\left[\sigma^{+}(a_k)+\sigma^{-}(a_k)\right]\leq z^{*}+\varepsilon$$

where z^* is the optimal value of the LP in step 3 and ε a very mall positive number.

THE UTASTAR ALGORITHM #8



Post-optimality analysis
(Jacquet-Lagrèze and Siskos, 1982)

UTASTAR vs UTA

- A comparison analysis between UTA and UTASTAR algorithms was done by Siskos and Yannacopoulos (1985) through a variety of experimental data.
- UTASTAR method has provided better results concerning a number of comparison indicators, like:
  1. The number of the necessary simplex iterations for arriving at the optimal solution.
  2. The Kendall's τ between the initial weak order and the one produced by the estimated model.
  3. The minimized criterion z (sum of errors) taken as the indicator of dispersion of the observations.

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #1

Criteria

1. price (in monetary units)
2. time of journey (in minutes)
3. comfort (possibility to have a seat)

Comfort scale

0 : no chance of seating

\+ : little chance of seating

++ : great chance of finding a seating place

+++ : seat assured

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #2

Multicriteria evaluation & Ranking

Means	Price (mu)	Time (min)	Comfort	Ranking
RER	3	10	+	1
METRO (1)	4	20	++	2
METRO (2)	2	20	0	2
BUS	6	40	0	3
TAXI	30	30	++	4

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #3

Discretization of scales and variable transformation

$$\left[g_{1*}, g_1^*\right] = [30,16,2]$$

$$\left[g_{2*}, g_2^*\right] = [40,30,20,10]$$

$$\left[g_{3*}, g_3^*\right] = [0,+,++,+++]$$

Using linear interpolation for the criterion g_1 , the value of each action is:

$$u[\mathbf{g}(\text{RER})] = 0.07u_1(16) + 0.93u_1(2) + u_2(10) + u_3(+)$$

$$u[\mathbf{g}(\text{METRO1})] = 0.14u_1(16) + 0.86u_1(2) + u_2(20) + u_3(++)$$

$$u[\mathbf{g}(\text{METRO2})] = u_1(2) + u_2(20) + u_3(0) = u_1(2) + u_2(20)$$

$$u[\mathbf{g}(\text{BUS})] = 0.29u_1(16) + 0.71u_1(2) + u_2(40) + u_3(0) = 0.29u_1(16) + 0.71u_1(2)$$

$$u[\mathbf{g}(\text{TAXI})] = u_1(30) + u_2(30) + u_3(+++) = u_2(30) + u_3(+++)$$

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #4

Discretization of scales and variable transformation

The following normalization conditions for the marginal value functions have been used:

$u_1(30) = u_2(40) = u_3(0) = 0$.

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #5

- The global value of the actions may be expressed in terms of the variables w_{ij} :

$$u[\mathbf{g}(\mathrm{RER})] = w_{11} + 0.93w_{12} + w_{21} + w_{22} + w_{23} + w_{31}$$
$$u[\mathbf{g}(\mathrm{METRO1})] = w_{11} + 0.86w_{12} + w_{21} + w_{22} + w_{31} + w_{32}$$
$$u[\mathbf{g}(\mathrm{METRO2})] = w_{11} + w_{12} + w_{21} + w_{22}$$
$$u[\mathbf{g}(\mathrm{BUS})] = w_{11} + 0.71w_{12}$$
$$u[\mathbf{g}(\mathrm{TAXI})] = w_{21} + w_{31} + w_{32} + w_{33}$$

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #6

- According to the second step of the UTASTAR algorithm, the following expressions are written, for each pair of consecutive actions in the ranking:

$$\Delta(RER,METRO1) = 0.07w_{12} + w_{23} - w_{32} - \sigma^{+}_{RER} + \sigma^{-}_{RER} + \sigma^{+}_{METRO1} - \sigma^{-}_{METRO1}$$

$$\Delta(METRO1,METRO2) = -0.14w_{12} + w_{31} + w_{32} - \sigma^{+}_{METRO1} + \sigma^{-}_{METRO1} + \sigma^{+}_{METRO2} - \sigma^{-}_{METRO2}$$

$$\Delta(METRO2,BUS) = 0.29w_{12} + w_{21} + w_{22} - \sigma^{+}_{METRO2} + \sigma^{-}_{METRO2} + \sigma^{+}_{BUS} - \sigma^{-}_{BUS}$$

$$\Delta(BUS,TAXI) = w_{11} + 0.71w_{12} - w_{21} - w_{31} - w_{32} - w_{33} - \sigma^{+}_{BUS} + \sigma^{-}_{BUS} + \sigma^{+}_{TAXI} - \sigma^{-}_{TAXI}$$

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #7

Linear programming formulation

w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}	Variables σ^+ and σ^-											RHS
0	0.07	0	0	1	0	-1	0	-1	1	1	-1	0	0	0	0	0	0	≥	0.05
0	0.14	0	0	0	1	1	0	0	0	-1	1	1	-1	0	0	0	0	=	0
0	0.29	1	1	0	0	0	0	0	0	0	0	-1	1	1	-1	0	0	≥	0.05
1	0.71	-1	0	0	-1	-1	-1	0	0	0	0	0	0	-1	1	1	-1	≥	0.05
1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	=	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	z	

Marginal value functions (optimal solution)

Price	Time	Comfort
$u_1(30)=0.00$	$u_2(40)=0.00$	$u_3(0)=0.00$
$u_1(16)=0.50$	$u_2(30)=0.05$	$u_3(+)=0.00$
$u_1(2)=0.50$	$u_2(0)=0.05$	$u_3(++)=0.00$
	$u_2(10)=0.10$	$u_3(+++)=0.4$

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #8

- The initial linear program has multiple optimal solutions, since $z^*=0$. In the post-optimality analysis step, the algorithm searches for more characteristic solutions, which maximize the weight of each criterion. Furthermore, in this particular case we have:

$$z^* = 0 \Leftrightarrow \sigma^+(a_k) = \sigma^-(a_k) = 0 \quad \forall\, k$$

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #9

Robustness/post-optimality analysis

w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}		RHS
0	0.07	0	0	1	0	-1	0	≥	0.05
0	0.14	0	0	0	1	1	0	=	0
0	0.29	1	1	0	0	0	0	≥	0.05
1	0.71	-1	0	0	-1	-1	-1	≥	0.05
1	1	1	1	1	1	1	1	=	1
1	1	0	0	0	0	0	0	[max]$u_1(g_1^*)$	
0	0	1	1	1	0	0	0	[max]$u_2(g_2^*)$	
0	0	0	0	0	1	1	1	[max]$u_3(g_3^*)$	

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #10

Post-optimality analysis and final solution

	w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}
[max]$u_1(g_1^*)$	0.7625	0.175	0	0	0.0375	0.025	0	0
[max]$u_2(g_2^*)$	0.05	0	0	0.05	0.9	0	0	0
[max]$u_3(g_3^*)$	0.3562	0.175	0	0	0.0375	0.025	0	0.4063
Average	0.3896	0.1167	0	0.0167	0.3250	0.0167	0	0.1354

A NUMERICAL EXAMPLE THE CHOICE OF A TRANSPORTATION MEAN #11

Global values

$$u[g(RER)] = 0.856$$
$$u[g(METRO1)] = 0.523$$
$$u[g(METRO2)] = 0.523$$
$$u[g(BUS)] = 0.473$$
$$u[g(TAXI)] = 0.152$$

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #12

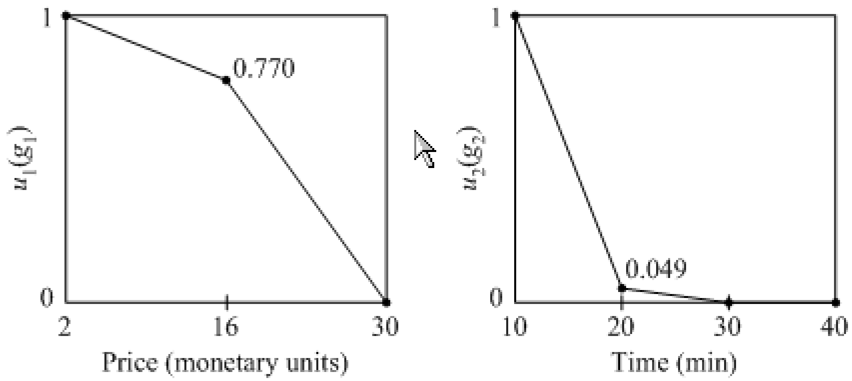
Additive value model (final solution)

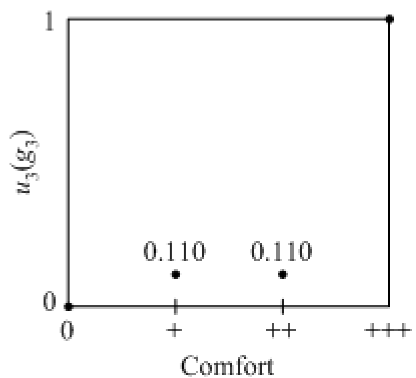
Price	Time	Comfort
$u_1(30) = 0.000$	$u_2(40) = 0.000$	$u_3(0) = 0.000$
$u_1(16) = 0.390$	$u_2(30) = 0.000$	$u_3(+) = 0.017$
$u_1(2) = 0.506$	$u_2(20) = 0.017$	$u_3(++) = 0.017$

$$u(\mathbf{g}) = 0.506u_1(g_1) + 0.342u_2(g_2) + 0.152u_3(g_{13})$$

A NUMERICAL EXAMPLE
THE CHOICE OF A TRANSPORTATION MEAN #13

Normalized marginal value functions





SOME VARIANTS OF UTA #1

Global preference in terms of pairwise comparisons

$$\begin{cases} [\min] F = \sum_{(a,b):a \succ b} \lambda_{ab} z_{ab} + \sum_{(a,b):a \sim b} \lambda_{ab} z_{ba} \\ \text{subject to} \\ \sum_{i=1}^{n} \left\{ u_i \left[g_i(a) \right] - u_i \left[g_i(b) \right] \right\} + z_{ab} \geq 0 \qquad \text{if } a \succ b \\ \sum_{i=1}^{n} \left\{ u_i \left[g_i(a) \right] - u_i \left[g_i(b) \right] \right\} + z_{ab} - z_{ba} = 0 \quad \text{if } a \sim b \ \left(\Rightarrow b \sim a \right) \\ u_i(g_i^{j+1}) - u_i(g_i^j) \geq s_i \qquad \forall i, j \\ \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ u_i(g_{i*}) = 0, \ u_i(g_i^j) \geq 0, \ z_{ab} \geq 0 \quad \forall i, j \text{ and } (a,b) \in R \end{cases}$$

λ_{ab} being a non negative weight reflecting a degree of confidence in the judgment between *a* and *b*.

SOME VARIANTS OF UTA #2

Maximizing Kendall's τ

$$\begin{cases} [\min]F = \sum_{(a,b)\in R} \gamma_{ab} \Leftrightarrow [\max]\tau(R,R') \\ \text{subject to} \\ \sum_{i=1}^{n} \left\{ u_i\left[g_i(a)\right] - u_i\left[g_i(b)\right] \right\} + M\cdot\gamma_{ab} \geq \delta & \forall (a,b)\in R \\ u_i(g_i^{j+1}) - u_i(g_i^{j}) \geq s_i & \forall i,j \\ \sum_{i=1}^{n} u_i(g_i^{*}) = 1 \\ u_i(g_{i*}) = 0,\ u_i(g_i^{j}) \geq 0 & \forall i,j \\ \gamma_{ab} = 0 \text{ or } 1 & \forall (a,b)\in R \end{cases}$$

where M is a large number.

SOME VARIANTS OF UTA #3

UTA II

- **First step**: The marginal value functions are built outside the UTA algorithm, by techniques such as:
  a) techniques based on MAUT theory and described by Keeney and Raiffa (1976), and Klein *et al*. (1985),
  b) MACBETH method (Bana e Costa and Vansnick, 1994, 1997; Bana e Costa *et al*., 2001),
  c) Quasi-UTA method by Beuthe *et al*. (2000), that uses "recursive exponential" marginal value functions, and
  d) MIIDAS system (see section 4) that combines artificial intelligence and visual procedures in order to extract the DM's preferences (Siskos *et al*., 1999).

SOME VARIANTS OF UTA #4

UTA II

- **First step**: The marginal value functions are built outside the UTA algorithm, by techniques such as:
  a) techniques based on MAUT theory and described by Keeney and Raiffa (1976), and Klein *et al*. (1985),
  b) the MACBETH method (Bana e Costa and Vansnick, 1994, 1997; Bana e Costa *et al*., 2001),
  c) the Quasi-UTA method by Beuthe *et al*. (2000), that uses “recursive exponential” marginal value functions, and
  d) the MIIDAS system (see section 4) that combines artificial intelligence and visual procedures in order to extract the DM’s preferences (Siskos *et al*., 1999).

SOME VARIANTS OF UTA #5

UTA II

- **Second step**: The DM is asked to give a global ranking of alternatives in a similar way as in the basic UTA method. From this information, the problem may be formulated via a LP, in order to **assess only the weighting factors** of the criteria (scaling constants of criteria).

Setting:

$$\Delta(a,b) = \sum_{i=1}^{n} p_i \left\{ u_i[g_i(a)] - u_i[g_i(b)] \right\} - \sigma^+(a) + \sigma^-(a) + \sigma^+(b) - \sigma^-(b)$$

SOME VARIANTS OF UTA #6

UTA II

Solving the following LP:

$$\begin{cases} [\min]F = \sum_{a \in A_R} \left[\sigma^+(a) + \sigma^-(a) \right] \\ \text{subject to} \\ \Delta(a,b) \geq \delta & \text{if } a \succ b \\ \Delta(a,b) = 0 & \text{if } a \sim b \\ \sum_{i=1}^{n} p_i = 1 \\ p_i \geq 0,\ \sigma^+(a) \geq 0,\ \sigma^-(a) \geq 0 & \forall a \in A_R,\ \forall i \end{cases}$$

SOME VARIANTS OF UTA #7

UTADIS

- The extension of the UTA method in the case of a discriminant analysis model was firstly discussed by Jacquet-Lagrèze and Siskos (1982). The aim is to infer *u* from assignment examples in the context of **problematic β**. In the presence of two classes, if the model is without errors, the following inequalities must hold:

$$\begin{cases} a \in A_1 \Leftrightarrow u\left[\mathbf{g}(a)\right] \geq u_o \\ a \in A_2 \Leftrightarrow u\left[\mathbf{g}(a)\right] < u_o \end{cases}$$

SOME VARIANTS OF UTA #8

UTADIS

with u_0 being the level of acceptance/rejection which must be found in order to distinguish the set of accepted actions called A_1 and the set of rejected actions called A_2.

- Introducing the error variables $\sigma(a)$, $a \in A_R$, the objective is to minimize the sum of deviations from the threshold u_0 for the ill classified actions. Hence, $u(\mathbf{g})$ can be estimated by means of the LP:

SOME VARIANTS OF UTA #9

UTADIS

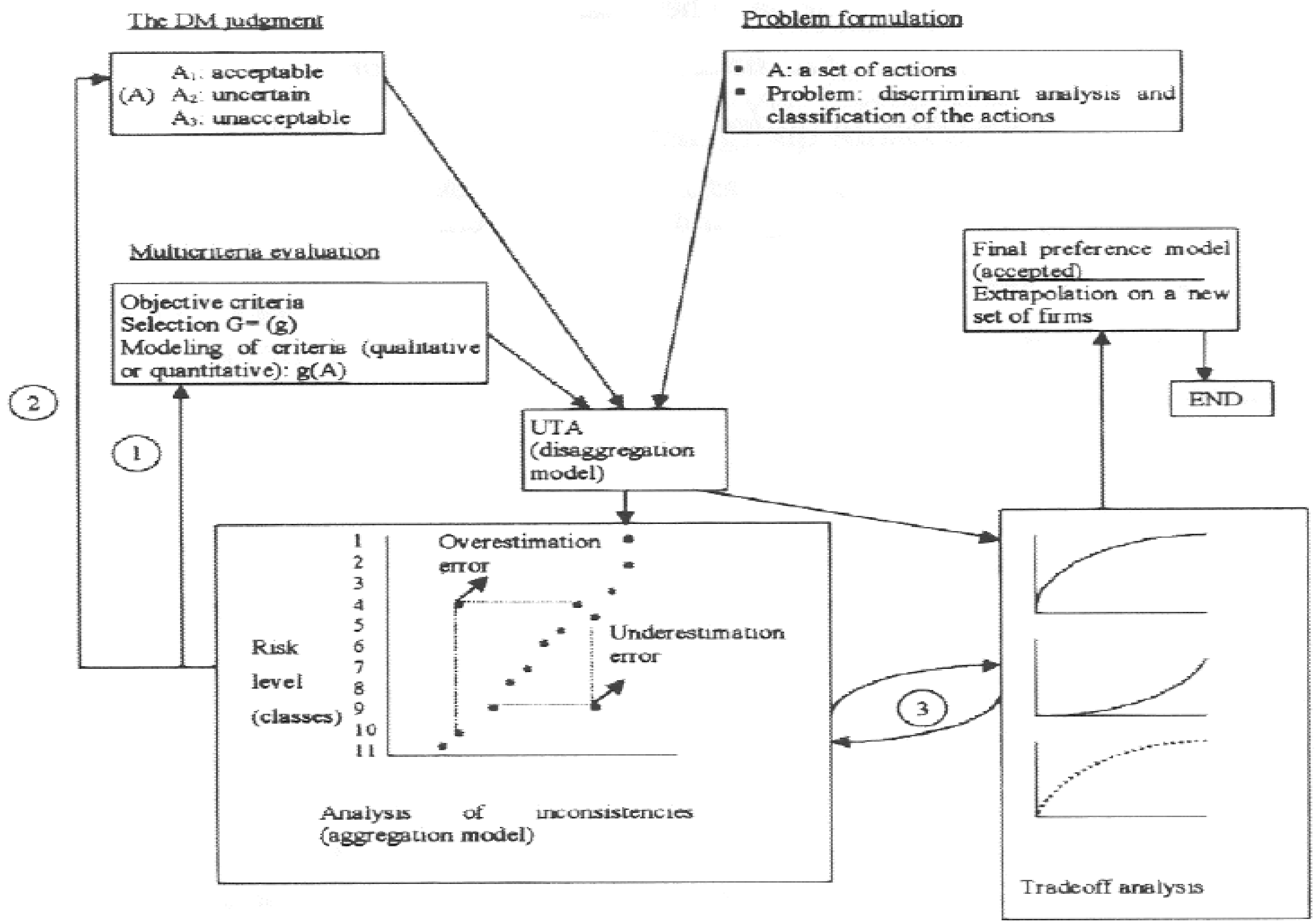
$$\begin{cases} [\min] F = \sum_{a \in A_R} \sigma(a) \\ \text{subject to} \\ \sum_{i=1}^{n} u_i\left[g_i(a)\right] - u_0 + \sigma(a) \geq 0 & \forall a \in A_1 \\ \sum_{i=1}^{n} u_i\left[g_i(a)\right] - u_0 - \sigma(a) \leq 0 & \forall a \in A_2 \\ u_i(g_i^{j+1}) - u_i(g_i^{j}) \geq s_i & \forall i \text{ and } j \\ \sum_{i=1}^{n} u_i(g_i^{*}) = 1 \\ u_i(g_{i*}) = 0,\ u_0 \geq 0,\ u_i(g_i^{j}) \geq 0,\ \sigma(a) \geq 0 & \forall a \in A_R,\ \forall i \text{ and } j \end{cases}$$

UTA-BASED DSS #1

- The disaggregation approach constitutes a basis for the interaction between the analyst and the DM, which includes:
  - the consistency between the assessed preference model and the global preference of the DM,
  - the assessed values (values, weights, utilities, …), and
  - the overall evaluation of potential actions (extrapolation output).

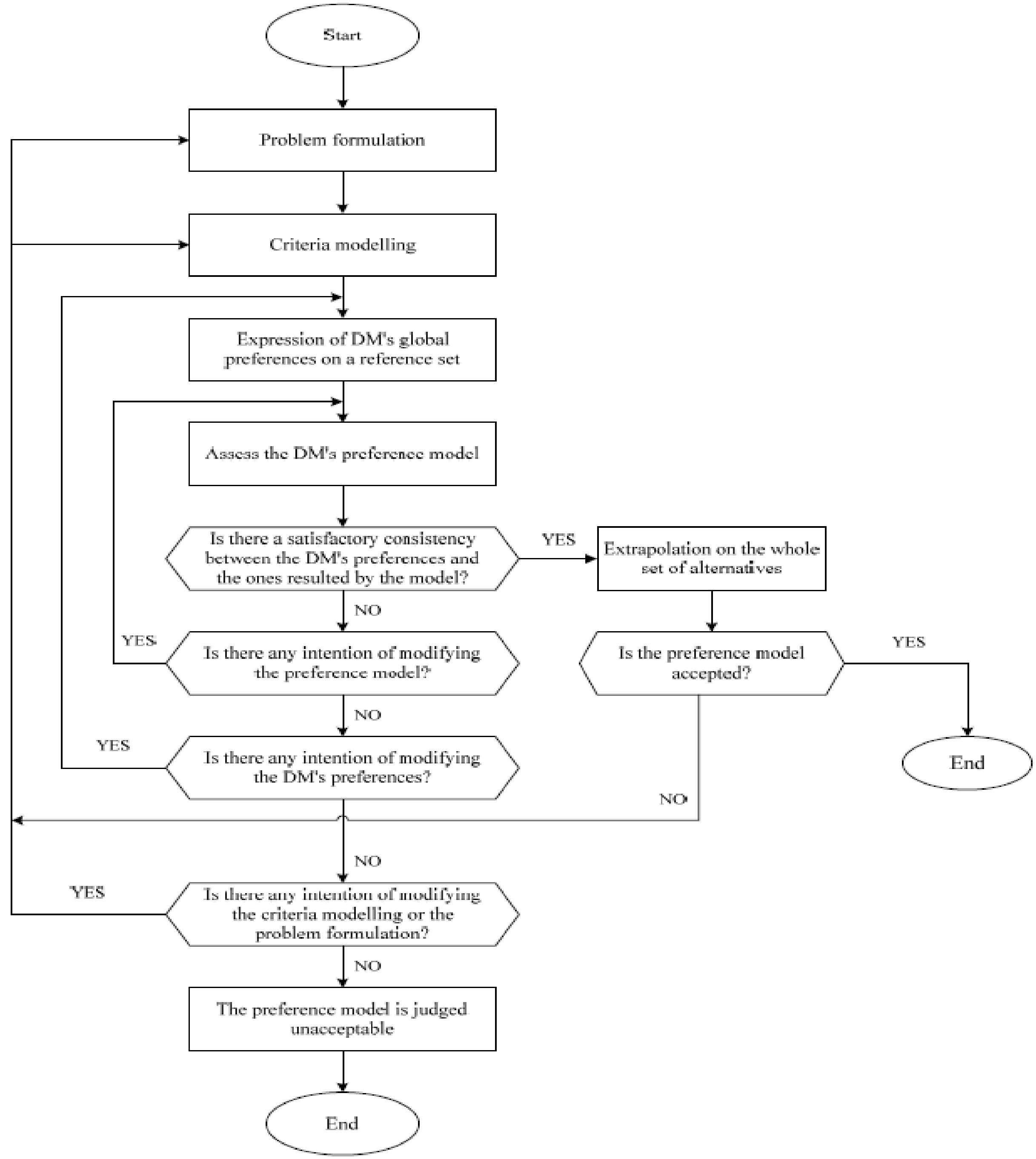
UTA-BASED DSS #2

The MINORA Principle



UTA-BASED DSS #3

Simplified decision support procest based on disaggregation approach (Jacquet-Lagrèze and Siskos, 2001)



UTA-BASED DSS #4

Decision support systems (DSSs) based on UTA

- **PREFCALC** system (Jacquet-Lagrèze, 1990) is a DSS for interactive assessment of preferences using holistic judgments. The interactive process includes the classical aggregation phase where the DM is asked to estimate directly the parameters of the model (i.e. weights, trade-offs, etc.), as well as the disaggregation phase where the DM is asked to express his/her holistic judgments (i.e. global preference order on a subset of the alternatives) enabling an indirect estimation of the parameters of the model.
- **MINORA** (Multicriteria Interactive Ordinal Regression Analysis) is a multicriteria interactive DSS with a wide spectrum of supported decision making situations (Siskos et al., 1993, 1994). The core of the system is based on the UTASTAR method and it uses special interaction techniques in order to guide the DM to reach a consistent preference system.
- **MIIDAS** (Multicriteria Interactive Intelligence Decision Aiding System) is an interactive DSS that implements the extended UTA II method (Siskos et al., 1999). In the first step of the decision-aiding process, the system assess the DM’s value functions, while in the next step, the system estimates the DM’s preference model from his/her global preferences on a reference set of alternative actions. The system uses Artificial Intelligence and Visual techniques in order to improve the user interface and the interactive process with the DM.

UTA-BASED DSS #5

Decision support systems (DSSs) based on UTA

- **UTA PLUS** software (Kostkowski and Slowinski, 1996; http://www.lamsade.dauphine.fr/english/software.html#uta+) is an implementation of the UTA method, which allows the user to modify interactively the marginal value functions within limits following from a sensitivity analysis of the formulated ordinal regression problem. During all these modifications, a friendly graphical interface helps the DM to reach an accepted preference model.
- **MUSTARD** (Multicriteria Utility-based Stochastic Aid for Ranking Decisions) is an interactive DSS developed by Beuthe and Scannella (1999), which incorporates several variants of the UTA method. The system provides several visual tools in order to structure the DM’s preferences to a specific problem (see also Siskos, 2002). The interactive process with the DM contains the following main steps: problem structuring, preference questionnaire, optimization solver-parameter computing, final results (full rankings and graphs).
- **MEDIATOR** system developed (Jarke *et al*., 1988; Shakun, 1988; Shakun, 1991), which is a negotiation support system based on Evolutionary Systems Design (ESD) and database-centered implementation. ESD visualizes negotiations as a collective process of searching for designing a mutually acceptable solution.

UTA-BASED DSS #6

Decision support systems (DSSs) based on UTA

- **MARKEX** system proposed by Siskos and Matsatsinis (1993), Matsatsinis and Siskos (1999), Matsatsinis and Siskos (2003). The system includes the UTASTAR algorithm and is used for the new product development process. It acts as a consultant for marketers, providing visual support to enhance understanding and to overcome lack of expertise.
- **AgentAllocator** system (Matsatsinis and Delias, 2003) implements the UTA II method in the task allocation problem. These problems are very common to any multi-agent system in the context of Artificial Intelligence. The system is an intelligent agent DSS, which allows the DM to model his/her preferences in order to reach and employ the optimal allocation plan.
- **FINEVA** system (Zopounidis *et al*., 1996) is a multicriteria knowledge-based DSS developed for the assessment of corporate performance and viability. The system implements multivariate statistical techniques (e.g. principal components analysis), expert systems technology, and the UTASTAR method to provide integrated support in evaluating the corporate performance.

UTA-BASED DSS #7

Decision support systems (DSSs) based on UTA

- **FINCLAS** system (Zopounidis and Doumpos, 1998) is a multicriteria DSS developed to study financial decision-making problems in which a classification (sorting) of the alternatives is required. The present form of the system is devoted to corporate credit risk assessment, and it can be used to develop classification models to assign a set of firms into predefined credit risk classes. The analysis performed by the system is based on the family of the UTADIS methods.
- **INVESTOR** system (Zopounidis and Doumpos, 2000b) is developed to study problems related to portfolio selection and management. The system implements the UTADIS method, as well as goal programming techniques to support portfolio managers and investors in their daily practice.
- **PREFDIS** system (Zopounidis and Doumpos, 2000c) is a multicriteria DSS developed to address classification problems. The system implements a series of preference disaggregation analysis techniques, namely the family of the UTADIS methods, in order to develop an additive utility function to be used for classification purposes.

UTA-BASED DSS #8

Decision support systems (DSSs) based on UTA

- **ADELAIS** system (Siskos and Despotis (1989) designed to decision-aid in multiobjective linear programming (MOLP) problems.
- **MUSA** software developed in order to measure and analyze customer satisfaction (Siskos *et al*., 1998; Grigoroudis and Siskos, 2002). The method is used for the assessment of a set of marginal satisfaction functions in such a way that the global satisfaction criterion becomes as consistent as possible with customer's judgments. Thus, the main objective of the method is the aggregation of individual judgments into a collective value function.